Code: 15A54101

R15

B.Tech I Year I Semester (R15) Supplementary Examinations November/December 2019

MATHEMATICS – I

(Common to all branches)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Solve $(x + y + 1)\frac{dy}{dx} = 1$.

(b) Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$.

(c) Find the envelop of the family of the lines $= mx + \sqrt{(1+m)^2}$, m being the parameter.

(d) Find the evolute of the parabola $y^2 = 4ax$.

(e) Evaluate $\int_0^2\int_0^x y\,dy\,dx$.

(f) Using cylindrical coordinates, find the volume of the cylinder with base radius *a*, and height *h*.

(g) Find the extema of $f(x, y) = a^2 - x^2 - y^2$.

(h) Verify Euler's theorem for $u(x, y, z) = xy + yz + zx$.

(i) If $R = xi + yj + zk$, show that $\nabla \cdot R = 3$ and $\nabla \times R = 0$.

(j) If $F = 3xyi - y^2 j$, then evaluate $\int F.dR$, where *C* is the curve in the *xy*-plane $y = 2x^2$ from (0, 0) to (1, 2).

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 Solve $(D^2 + 4D + 3)y = e^x\, cos2x - cos3x - 3x^2$.

OR

3 Find the equation of the tangent at any point (*x*, *y*) to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, show that the portion of the tangent intercepted between the axes is of constant length.

UNIT – II

4 (a) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$.

(b) A rectangle sheet of metal of length 6 meters is given. Four equal squares are removed from the corners. The sides of this sheet are now turned up to form an open rectangular box. Find approximately, the height of the box, such that the volume of the box is maximum.

OR

5 The deflection of a strut of length *l* with one end built-in and the other end subjected to the end thrust *p*, satisfies $\frac{d^2y}{dx^2} + ay = \frac{a^2R}{p}(l - x)$. Find the deflection *y* of the strut at a distance x from the built-in end.

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UNIT – III

6 (a) A rectangular box open at the top is to have a volume of 32 cft. Find the dimensions of the box requiring least material for its construction.

(b) Find the points on the surface $z^2 = xy + 1$ nearest to the origin.

OR

7 (a) Discuss the maxima and minima of $f(x, y) = \sin x \sin y \sin (x + y)$.

(b) Find the Taylor's series expansion of $tan^{-1}(y/x)$ about (1, 1).

UNIT – IV

8 Evaluate $\iiint xyz \; dx\, dy\, dz$ over the positive of the sphere $x^2 + y^2 + z^2 = a^2$.

OR

9 Find the volume bounded by the cylinders $x^2 + y^2 = 4$, $y + z = 4$ and $z = 0$.

UNIT – V

10 (a) Verify Stoke's theorem for $\overline{A} = y^2 i + xyj - xzk$ where S is the hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$.

(b) Evaluate $\int_S F.ds$ where $F = 4xi = 2y^2 j + z^2 k$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

OR

11 Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2 dy]$, where C is bounded by $y = x$ and $y = x^2$.

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