Code: 15A54201 **R15**

B.Tech I Year II Semester (R15) Supplementary Examinations December 2019

MATHEMATICS – II

(Common to all)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) State and prove first shifting theorem.

(b) Evaluate $\int_0^\infty te^{-2t}\sin t\,dt$.

(c) If $f(x) = x + x^3$ in $(-\pi, \pi)$, find the Euler's coefficients a_0, a_n.

(d) State the conditions for f(x) to have Fourier series expansion.

(e) Find the Fourier sine transform of the function $f(x) = 5e^{-2x} + 2e^{-5x}$.

(f) Find the Fourier transform of the function $f(x) = \begin{cases} x^2, |x| \le a \\ 0, \quad |x| > a \end{cases}$.

(g) Write down possible solutions of the Laplace equation.

(h) A rod 20 cms long has its ends A and B kept at 30°C and 70°C respectively until steady state is prevailed. Determine the steady state temperature of the rod.

(i) Find $Z\left[\frac{1}{n-1}\right]$.

(j) State final value theorem on Z-transform.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Use convolution theorem to find $L^{-1}\left(\frac{1}{s^2(s+1)^2}\right)$.

(b) Find the Laplace transform of the square wave function of period α defined as:

$$f(t) = \begin{cases} 1, when \;\; 0 < t < \alpha/2 \\ -1, when \;\; \dfrac{\alpha}{2} < t < \alpha \end{cases}$$

OR

3 Solve $y''' + 2y'' - y' - 2y = 0$ given y(0) = 0, y'(0) = 0 and y''(0) = 6.

UNIT – II

4 Find the complex form of the Fourier series of $f(x) = e^{-x}$ in $-1 \le x \le 1$

OR

5 Obtain the Half Range cosine series of f(x) = x in 0<x<1. Hence deduce that $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots = \frac{\pi^4}{96}$.

UNIT – III

6 Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$.

OR

7 (a) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

(b) Using Parseval's identity, evaluate $\int_0^\infty \frac{x^2\,dx}{(x^2+1)^2}$.

Contd. in page 2

UNIT – IV

8 A tightly stretched string of length l with fixed end is initially in its equilibrium position. It is set vibrating by giving each point a velocity $V_0 \sin^3(\pi x/l)$. Determine the displacement function y(x, t).

OR

9 An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π; this end is maintained at a temperature u_0 at all points and other edges are at zero temperature. Determine the temperature at any point of the plate in the steady state.

UNIT – V

10 (a) Using convolution theorem, find the inverse Z-transform of $\frac{z^2}{(z-1)(z-3)}$.

(b) Find $z^{-1}\frac{2z}{(z-1)(z^2+1)}$, by partial fraction method.

OR

11 Solve: $y_{n+2} + 6y_{n+1} + 9y_n = 2^n \text{ given that } y_0 = 0 \text{ and } y_1 = 0$, using Z-transforms.

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