Code: 15A54201

R15

B.Tech I Year II Semester (R15) Regular & Supplementary Examinations May/June 2019

MATHEMATICS – II

(Common to all)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) State sufficient conditions for the existence of Laplace transform.

(b) Find $L^{-1}\left[\cot^{-1}\left(\frac{s}{2}\right)\right]$.

(c) Find the half range sine series for $f(x) = 1\ in\ (0,\pi)$.

(d) Define complex form of Fourier series.

(e) Find the Fourier cosine transform of the function $f(x) = \begin{cases} cosx, & 0 < x < a \\ 0, & x > a \end{cases}$.

(f) State Fourier integral theorem.

(g) A rod 30 cms long has its ends A and B kept at 20°C and 80°C respectively until steady state is prevailed. Determine the steady state temperature of the rod.

(h) Form the PDE by eliminating the arbitrary constant from $z = (x^2 + a^2)(x^2 + b^2)$.

(i) State initial value theorem on z-transform.

(j) Find $Z\left[\frac{1}{n+1}\right]$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Use convolution theorem to find $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$.

(b) Find the Laplace transform of the triangular wave of period 2a given by:

$$f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 21 \end{cases}$$

OR

3 Solve: $y'' - 3y' + 2y = 4t + e^{3t}, when\ y(0) = 1\ and\ y'(0) = -1$ by using Laplace transform method.

UNIT – II

4 Obtain the Fourier series for $f(x) = x^2\ in - \pi < x < \pi$. Using the two values of y show that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$

OR

5 Obtain the Fourier series expansion for f(x), if $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.

UNIT – III

6 Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3}\cos\frac{x}{2}\,dx$.

OR

7 (a) Using Parseval's identity, evaluate $\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)}$, using Fourier transform method.

(b) Find the Fourier sine transform of the function $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$.

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UNIT – IV

8 A string is stretched and fastened to two points x = 0 and x = l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ form which it is released at time t = 0. Find the displacement of any point on the string at a distance of x from one end at time t.

OR

9 A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. If the temperature of the short edge y = 0 is given by:

$$u = \begin{cases} 20x & for\ 0 \le x \le 5 \\ 20(10-x) & for\ 5 \le x \le 10 \end{cases}$$

And the two long edges x = 0, x = 10 as well as the other short edge are kept at 0^{o}C. Find the steady state temperature at any point (x, y).

UNIT – V

10 (a) Using convolution theorem, find the inverse Z-transform of $\frac{z^2}{(z-a)(z-b)}$.

(b) Find $z^{-1}\frac{2z^2+3z}{(z+2)(z-4)}$, by partial fraction method.

OR

11 Solve: $u_{n+2} + 4u_{n+1} + 3u_n = 3^n\ given\ that\ u_0 = 0\ \ and\ u_1 = 1$, using Z-transforms.

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