Roll No. $\square$ Total No. of Pages : 02
Total No. of Questions: 18

B.Tech. (Automation \& Robotics) (2012 \& Onward) (Sem.-3)<br>MATHEMATICS-III<br>Subject Code: BTAR-301<br>M.Code : 63001

Time : 3 Hrs.
Max. Marks : 60

## INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.

## SECTION-A

1. If Laplace transform of $f(t)$ is $\mathbf{F}(\mathrm{s})$ then find Laplace Transform $L\left(\int_{0}^{t} f(t) d t\right)$, if it exists.
2. Find Inverse Laplace Transform of $\frac{2 s+6}{s^{2}+4}$.
3. Write down the Bessel's differential equation of order ' $n$ '.
4. Define error function.
5. Express the following in terms of Lagendre polynomials $1-x+x^{2}$
6. Show that cosz is an analytic function.
7. Define a conformal mapping.
8. Evaluate $\int_{C} \frac{1+z^{2}}{z-2} d z, C:|z|=3$
9. Evaluate the integral $\int_{0}^{3+i} z^{2} d z$ along the line $y=x / 3$
10. Expand $\log (1+z)$ about $z=0$.

## SECTION-B

11. Solve the differential equation using Method of Laplace transform

$$
\frac{d^{2} y}{d t^{2}}+2 \frac{d y}{d t}+5 y=\sin 2 t, \text { given that } y(0)=2, y^{\prime}(0)=-4
$$

12. Prove that $(2 n+1) x \mathrm{P}_{n}(x)=(n+1) \mathrm{P}_{n+1}(x)+n \mathrm{P}_{n-1}(x)$
13. Show that the function defined by $f(z)=\left\{\begin{array}{cc}\frac{x^{2} y^{3}(x+i y)}{x^{6}+y^{10}} & z=0 \\ 0, & z \neq 0\end{array}\right.$, is analytic at origin, even though $f^{\prime}(0)$ does not exist.
14. If the potential function is $\log \left(x^{2}+y^{2}\right)$, find the flux function and complex potential function.
15. Show that the transformation $w=\frac{z-i}{z+i}$ maps the real axis in the z-plane onto the circle $|w|=1$.

## SECTION-C

16. (a) Define unit step function and find its Laplace transform.
(b) Prove that $\frac{d}{d x}\left[x^{-n} J_{n}(x)\right]=-x n^{n} f_{n+1}^{n}(x)$
17. Solve by applying Frobenius method: $9 x(1-x) \frac{d^{2} y}{d x^{2}}-12 \frac{d y}{d x}+4 y=0$
18. Using Contour integration, evaluate the integral $\int_{0}^{\infty} \frac{d x}{x^{4}+1}$

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.

