Roll No. | | | | | | | | | | | | | Total No. of Pages : 02

Total No. of Questions : 18

B.Tech. (Automation & Robotics) (2018 Batch) (Sem.–3)

MATHEMATICS-III

Subject Code : BTAR-303-18

M.Code : 76502

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

Write briefly :

1. Find the Fourier series of the function $f(x) = |x|$ over the interval $[-2, 2]$.
2. Find Laplace transform of $e^{-t} \sin^2 t$.
3. State and prove Second Shifting Property for Laplace transform.
4. Find inverse Laplace transform of $\frac{2s-3}{s^2+4s+13}$.
5. Express sum of Legendre polynomials $8P_4(x) + 2P_3(x) + P_0(x)$ in terms of powers of x.
6. For Legendre polynomial $P_n(x)$, show that $P_n(-x) = (-1)^n P_n(x)$
7. Form a partial differential equation by eliminating arbitrary function f from the relation $z = y^2 = 2f\left(\frac{1}{x} + \log y\right)$.
8. Solve $z(xp - yq) = y^2 - x^2$.
9. Show that the function $u(x, y) = 2x + y^3 - 3x^2y$ is harmonic.
10. State Cauchy Integral Theorem.

SECTION-B

11. Find the Fourier series expansion of the function

$$f(x)=\begin{cases}0, & for -\pi \le x < 0\\ 1, & for\ 0 \le x \le \pi\end{cases}.$$ Deduce that $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+.....$

12. State and prove Convolution Theorem for Laplace transform.

13. For Legendre polynomial $P_n(x)$, show that :

$$\int_{-1}^{1} P_m(x)\ P_n(x)\,dx=\frac{2}{2n+1},\ \text{for } m = n.$$

14. Solve by Charpit's method $z = p^2x + q^2y$.

15. Evaluate $\oint_C \frac{3z+5}{z^2+2z}\,dz$, C : $|z| = 1$.

SECTION-C

16. a) Using Laplace transform, solve $y'' - 6y' + 9y = e^{3t}t^2$, $y(0) = 2$, $y'(0) = 6$.

    b) Find inverse Laplace transform of $\frac{2s+1}{(s+2)^2(s-1)^2}$.

17. a) Solve Legendre differential equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$.

    b) Using the method of separation of variables, solve

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2},\ u(x,0)=x^2,\ u(0,t)=u(2\pi,t)=0.$$

18. a) Find all Taylor and Laurent series expansions of $f(z)=\frac{1}{z^2+1}$ about the point $z = i$.

    b) Compute the residues at the singular points $z = 1, -2$ of

$$f(z)=\frac{1+z+z^2}{(z-1)^2(z+2)}$$

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.