## Answer any five questions All questions carry equal marks

1.a) Show that $(\mathrm{p} \wedge \mathrm{q}) \rightarrow(\mathrm{p} \vee \mathrm{q})$ is a tautology using logical equivalences and truth table.
b) Let p and q be the propositions.
p :You drive over 65 miles per hour.
q :You get a speeding ticket.
Write these propositions using p and q and logical connectives (including negations).
i) You do not drive over 65 miles per hour.
ii) You drive over 65 miles per hour, but you do not get a speeding ticket.
iii) You will get a speeding ticket if you drive over 65 miles per hour.
iv) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
v) Driving over 65 miles per hour is sufficient for getting a speeding ticket. [7+8]
2.a) Let p and q be the propositions.
p :I bought a lottery ticket this week,
q :I won the million dollar jackpot.
Express each of these propositions as an English sentence.
i) $p \vee q$
ii) $p \rightarrow q$
iii) $\mathrm{p} \leftrightarrow \mathrm{q}$
iv) $\neg \mathrm{p} \rightarrow \neg \mathrm{q} \quad \mathrm{v}) \neg \mathrm{p} \leftrightarrow \neg \mathrm{q}$
b) Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book".
3.a) Let $A=\{0,2,4,6,8 \perp B=\{0,1,2,3,4\}$, and $C=\{0,3,6,9\}$. What are $(A \cup B \cup C)$ and ( $\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}$ )?
b) Draw the Venn diagrams for each of these combinations of the sets $\mathrm{A}, \mathrm{B}$, and C .
i) $A \cap(B-C)$
ii) $(\mathrm{A} \cap \mathrm{B}) \cup(\mathrm{A} \cap \mathrm{C})$
c) Define one-to-one and on-to functions.
4.a) Find fog and gof, where $f(x)=2 x+1$ and $g(x)=x^{2}+1$, are functions from $R$ to $R$.
b) How can we produce the terms of a sequence if the first 10 terms are $5,11,17,23,29,35$, $41,47,53,59$ ?
c) $\quad$ Let $R=\{(a, b),(b, c),(c, d),(d, e),(c, a),(a, c),(e, b)\}$ be a relation on the set $A=\{a, b, c, d, e\}$. Find the transitive closure of the relation $R$.
5.a) Determine whether each of the functions $2^{n+1}$ and $2^{2 n}$ is $\mathrm{O}\left(2^{n}\right)$.
b) Give a recursive algorithm for computing the greatest common divisor of two nonnegative integers a and b with $\mathrm{a}<\mathrm{b}$.
6.a) Find the probability that a hand of five cards in poker contains four cards of one kind.
b) What is the probability that a positive integer selected at random from the set of positive integers not exceeding 100 is divisible by either 2 or 5 ?
c) Define conditional probability.
7.a) Find the solution to the recurrence relation: $a_{n}=6 a_{n-1}-11 a_{n-2}+6 a_{n-3}$ with the initial conditions $\mathrm{a}_{0}=2, \mathrm{a}_{1}=5$, and $\mathrm{a}_{2}=15$.
b) Use generating functions to find the number of k -combinations of a set with n elements. Assume that the binomial theorem has already been established.
8. Devise the algorithms for DFS and BFS and explain the differences between them with an illustrative example.

