R15

Code No: 821AA

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

MCA I Semester Examinations, August - 2017

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Time: 3hrs **Max.Marks:75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART - A

5 × 5 Marks = 25

1.a) What do you mean by a well-formed formula? Give examples of formulas that are well-formed and not well-formed. [5]

b) What do you mean by a lattice? List the properties of a lattice. [5]

c) What are the two basic counting principles. [5]

d) Give the generating functions for the sequences $C(k,n)$, a^n, $(-1)^n$ and n. [5]

e) Is there a graph with degree sequence (1,3,3,3,5,6,6)? Justify your answer. [5]

PART - B

5 × 10 Marks = 50

2.a) Show the following equivalences:

i) $A \rightarrow (P \vee C) \Leftrightarrow (A \wedge \neg P) \rightarrow C$

ii) $(P \rightarrow C) \wedge (Q \rightarrow C) \Leftrightarrow (P \vee Q) \rightarrow C$

b) Show that the following premises are inconsistent:

i) If Jack misses many classes through illness, then he fails high school.
ii) If Jack fails high school, then he is uneducated.
iii) If Jack reads a lot of books, then he is not uneducated.
iv) Jack misses many classes through illness and reads a lot of books. [5+5]

OR

3.a) Obtain a principal conjunctive normal form of each of the following formulas:

i) $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$

ii) $P \rightarrow (P \wedge (Q \rightarrow P))$

b) Show that $(x)(P(x) \rightarrow Q(x)) \wedge (x)(Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x))$ [5+5]

4.a) Let $X = \{1,2,\ldots,7\}$ and $R = \{(x,y) \mid x - y$ is divisible by $3\}$. Show that R is an equivalence relation. Draw the graph of R.

b) Show that in a group $(G,*)$, if for any a, b $\in$ G, $(a*b)^2 = a^2 * b^2$, then $(G,*)$ must be abelian. [5+5]

OR

5.a) Let $f(x) = x+2, g(x) = x-2,$ and $h(x) = 3x$ for $x \in R,$ where R is the set of real numbers. Find $g \circ f, f \circ g, f \circ f, g \circ g, f \circ h \circ g$.

b) Find all the subgroups of $(Z_{12}, +_{12})$ and $(Z_7^*, \times_7)$ [5+5]

6.a) How many ways are there to distribute 10 balls into 6 boxes with atmost 4 balls in the first 2 boxes if:
i) The balls are indistinguishable
ii) The balls are distinguishable

b) Verify that $C(n+3,r)-3C(n+2,r)+3C(n+1,r)-C(n,r)=C(n,r-3)$ [5+5]

OR

7.a) Find the number of integral solutions for the following:
i) $x_1+x_2+x_3+x_4+x_5=10$ where $x_i \geq 0$
ii) $x_1+x_2+x_3+x_4=50$, where $x_1 \geq -4$, $\boldsymbol{x_2} \geq 7$, $\boldsymbol{x_3} \geq -14$, $x_4 \geq 10$

b) Determine the coefficient of x^5 in $(a+bx+cx^2)^{10}$ and $(x-7y)^{15}$. [5+5]

8.a) Build a generating function for $a_r =$ the number of integral solutions to the equation $x_1+x_2+x_3=r$
i) $0 \leq x_i \leq 3$ for each i
ii) $2 \leq x_i \leq \boldsymbol{5}$ for each i

b) Write a generating function for a_n**,** the number of ways of obtaining the sum n when tossing 9 distinguishable dice. Then find a_{25}. [5+5]

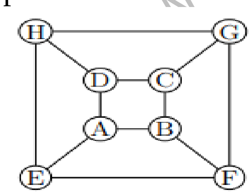
OR

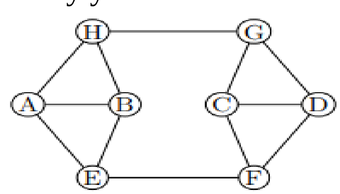
9.a) Solve the following recurrence relations using the characteristic roots:
i) $a_n-3a_{n-1}-4a_{n-2}=0$ for $n \geq 2$ and $a_0=a_1=1$.
ii) $a_n-4a_{n-1}-12a_{n-2}=0$ for $n \geq 2$ and $a_0=4$, $a_1=16/3$. [5+5]

b) Write the general form of a particular solution a_n^P to the following recurrence relations:
i) $a_n-7a_{n-1}+12a_{n-2}=n$
ii) $a_n-7a_{n-1}+12a_{n-2}=2^n$ [5+5]

10.a) Demonstrate with an example breadth-first search algorithm.

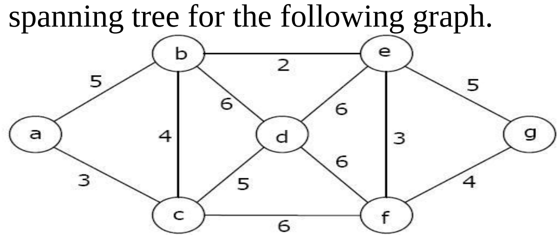
b) Are the graphs shown below isomorphic? Justify your answer. [5+5]





OR

11.a) Obtain the minimal spanning tree for the following graph.



b) Draw a full regular tree of degree 2 and height 3. [5+5]

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