0166

C

Total No. of Questions—24

Total No. of Printed Pages—4

Regd. No. | 1 | 8 | 6 | 0 | 1 | 2 | 3 | 7 | 8 | 3 |

Part III

MATHEMATICS, Paper - I(A)

(English Version)

Time : 3 Hours] *[Max. Marks : 75*

Note :—This question paper consists of three Sections A, B and C.

SECTION A

I. Very Short Answer Type Questions : 10×2=20

(*i*) Answer **ALL** questions.

(*ii*) Each question carries **TWO** marks.

1. If $a, b \in R$, $f : R \to R$ defined by $f(x) = ax + b$ $(a \neq 0)$, find f^{-1}.

2. Find the domain of real valued function :

$$f(x) = \sqrt{4x - x^2}.$$

3. If

$$A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$$

and $A^2 = 0$, then find the value of 'k'.

4. Find the rank of the following matrix :

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

5. Find the vector equation of the line joining the points $2\bar{i} + \bar{j} + 3\bar{k}$ and $-4\bar{i} + 3\bar{j} - \bar{k}$.

6. If $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$ and $\bar{b} = 4\bar{i} + m\bar{j} + n\bar{k}$ are collinear vectors, then find 'm' and 'n'.

7. If $\bar{a} = 2\bar{i} - 3\bar{j} + \bar{k}$ and $\bar{b} = \bar{i} + 4\bar{j} - 2\bar{k}$, then find $(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})$.

8. Prove that :

$$\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ.$$

9. Prove that :

$$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0.$$

10. Show that :

$$(\cosh x + \sinh x)^n = \cosh(nx) + \sinh(nx),$$

for any $n \in R$.

SECTION B

11. Short Answer Type Questions : 5×4=20

(*i*) Answer **ANY FIVE** questions.

(*ii*) Each question carries **FOUR** marks.

11. If

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

is a non-singular matrix, then prove that $A^{-1} = \frac{\text{Adj } A}{\det A}$.

12. If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors, then prove that :

$$-\bar{a} + 4\bar{b} - 3\bar{c}, 3\bar{a} + 2\bar{b} - 5\bar{c}, -3\bar{a} + 8\bar{b} - 5\bar{c}, -3\bar{a} + 2\bar{b} + \bar{c}$$

are coplanar.

13. For any two vectors $\bar{a}$ and $\bar{b}$, show that :

$$\left(1+|\bar{a}|\right)^2\left(1+|\bar{b}|\right)^2 = \left|1-\bar{a}\cdot\bar{b}\right|^2 + \left|\bar{a}+\bar{b}+\bar{a}\times\bar{b}\right|^2.$$

14. Prove that :

$$\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4.$$

15. Solve the equation :

$$\sin x + \sqrt{3}\cos x = \sqrt{2}.$$

16. Prove that :

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}.$$

17. Prove that :

$$\cot A + \cot B + \cot C = \frac{a^2+b^2+c^2}{4\Delta}.$$

SECTION C

III. Long Answer Type Questions : 5×7=35

(*i*) Answer **ANY FIVE** questions.

(*ii*) Each question carries **SEVEN** marks.

18. Let $f : A \to B$, I_A and I_B be identity functions on A and B respectively, then prove that $f \circ I_A = f = I_B \circ f$.

19. Show that $49^n + 16n - 1$ is divisible by 64 for all positive integers 'n'.

20. Show that :

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \left(a^3+b^3+c^3-3abc\right)^2.$$

21. Solve the following system of equations by Cramer's rule :

$$2x - y + 3z = 8$$
$$-x + 2y + z = 4$$
$$3x + y - 4z = 0.$$

22. If $\bar{a} = \bar{i} - 2\bar{j} + 3\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} + \bar{k}$, $\bar{c} = \bar{i} + \bar{j} + 2\bar{k}$, then find $\left|(\bar{a} \times \bar{b}) \times \bar{c}\right|$ and $\left|\bar{a} \times (\bar{b} \times \bar{c})\right|$.

23. If A + B + C = 2s, then prove that :

$$\cos(s - A) + \cos(s - B) + \cos(s - C) + \cos s = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}.$$

24. If P_1, P_2, P_3 are altitudes drawn from vertices A, B, C to the opposite sides of a triangle respectively, then show that :

(*i*) $$\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} = \frac{1}{r}$$

(*ii*) $$P_1P_2P_3 = \frac{(abc)^2}{8R^3} = \frac{8\Delta^3}{abc}.$$