

0193
TS**A**

Total No. of Questions – 24

Total No. of Printed Pages – 4

Regd.

No.

1	8	6	0	1	2	3	4	5	2
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Part - III
MATHEMATICS, Paper-I(B)
(English Version)

Time : 3 Hours]

[Max. Marks : 75

Note : This question paper consists of **three** sections **A, B** and **C**.**SECTION – A****10 × 2 = 20****I. Very short answer type questions :**(i) Attempt **all** questions.(ii) Each question carries **two** marks.1. Find the value of x , if the slope of the line passing through $(2, 5)$ and $(x, 3)$ is 2.2. Transform the equation $x + y + 1 = 0$ into normal form.3. Find the ratio in which the xz -plane divides the line joining $A(-2, 3, 4)$ and $B(1, 2, 3)$.4. Find the intercepts of the plane $4x + 3y - 2z + 2 = 0$ on the co-ordinate axes.5. Compute $\lim_{x \rightarrow 0} \left(\frac{\sin ax}{\sin bx} \right)$, $b \neq 0$, $a \neq b$



6. Compute $\lim_{x \rightarrow \pi/2} \left(\frac{\cos x}{x - \pi/2} \right)$

7. If $y = \frac{a-x}{a+x}$ ($x \neq -a$), find $\frac{dy}{dx}$

8. If $y = (\cot^{-1} x^3)^2$, find $\frac{dy}{dx}$

9. If the increase in the side of a square is 2%, then find the approximate percentage of increase in its area.



$$A = 10000$$

$$100 \times 102 = 10200$$

10. Find the value of C in Lagrange's mean value theorem for the function $f(x) = x^2 - 1$ on $[2, 3]$.

$$18L$$

$$18L$$

$$204$$

$$102 \times 2$$

$$10200$$

$$10200$$

$$20400$$

$$2104 - 1000$$

$$104$$

$$502$$

$$10040$$

SECTION - B

$$5 \times 4 = 20$$

Short answer type questions :

(i) Attempt any **five** questions.

(ii) Each question carries **four** marks.

11. Find the locus of the third vertex of a right angled triangle, the ends of whose hypotenuse are $(4, 0)$ and $(0, 4)$.

12. When the axes are rotated through an angle $\frac{\pi}{6}$, find the transformed equation of $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$.

13. Find the value of k, if the lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent.

14. Find the real constants a, b , so that the function f given by

$$f(x) = \begin{cases} \sin x, & \text{if } x \leq 0 \\ x^2 + a, & \text{if } 0 < x < 1 \\ bx + 3, & \text{if } 1 \leq x \leq 3, \end{cases} \quad f(x) = -3 \text{ if } x > 3$$

is continuous on \mathbb{R} .

15. Find the derivative of $x \sin x$ from the first principle.

16. Show that at any point (x, y) on the curve $y = b e^{x/a}$, the length of the sub-tangent is a constant and the length of the subnormal is $\frac{y^2}{a}$.

17. A particle is moving along a line according to $S = f(t) = 4t^3 - 3t^2 + 5t - 1$ where S is measured in metres and t is measured in seconds. Find the velocity and acceleration at time t . At what time the acceleration is zero?

SECTION - C

5 × 7 = 35

Long answer type questions :

- (i) Attempt any **five** questions.
- (ii) Each question carries **seven** marks.

18. Find the circumcenter of the triangle whose vertices are $(1, 3)$, $(-3, 5)$ and $(5, -1)$.
19. If the equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines, then show that the angle θ between the lines is given by

$$\cos \theta = \frac{|a + b|}{\sqrt{(a - b)^2 + 4h^2}}$$

20. Show that the lines joining the origin to the points of intersection of the curve

$x^2 - xy + y^2 + 3x + 3y - 2 = 0$ and the straight line $x - y - \sqrt{2} = 0$ are mutually perpendicular.

21. Find the angle between two diagonals of a cube.

22. If $y = \tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right]$ for $0 < |x| < 1$, find $\frac{dy}{dx}$.

23. Show that the equation of the tangent to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ ($a \neq 0, b \neq 0$) at the point (a, b) is $\frac{x}{a} + \frac{y}{b} = 2$.

24. From a rectangular sheet of dimensions $30 \text{ cm} \times 80 \text{ cm}$ four equal squares of side $x \text{ cm}$ are removed at the corners, and the sides are then turned up so as to form an open rectangular box. Find the value of x , so that the volume of the box is the greatest.

Handwritten solution for Q24:

$$V = x(80-2x)(30-2x)$$

$$\frac{dV}{dx} = (80-2x)(30-2x) + x(-2)(30-2x) + x(80-2x)(-2)$$

$$= (80-2x)(30-2x) - 2x(30-2x) - 2x(80-2x)$$

$$= (80-2x)(30-2x) - 2x(30+80-2x)$$

$$= (80-2x)(30-2x) - 2x(110-2x)$$

$$= (80-2x)(30-2x) - 220x + 4x^2$$

$$= 2400 - 160x - 60x + 4x^2 - 220x + 4x^2$$

$$= 2400 - 440x + 8x^2$$

$$\frac{dV}{dx} = 0 \Rightarrow 2400 - 440x + 8x^2 = 0$$

$$8x^2 - 440x + 2400 = 0$$

$$x^2 - 55x + 300 = 0$$

$$(x-20)(x-35) = 0$$

$$x = 20 \text{ or } x = 35$$

Since $x = 35$ is not possible (as $35 > 30$), $x = 20$ is the only possible value.