0193
TS

A

Total No. of Questions – 24
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Part - III
MATHEMATICS, Paper-I(B)
(English Version)

Time : 3 Hours] **[Max. Marks : 75**

Note : This question paper consists of **three** sections **A**, **B** and **C**.

SECTION – A 10 × 2 = 20

I. Very short answer type questions :

(i) Attempt **all** questions.

(ii) Each question carries **two** marks.

1. Find the value of x, if the slope of the line passing through (2, 5) and (x, 3) is 2.

2. Transform the equation $x + y + 1 = 0$ into normal form.

3. Find the ratio in which the xz-plane divides the line joining A (–2, 3, 4) and B (1, 2, 3).

4. Find the intercepts of the plane $4x + 3y - 2z + 2 = 0$ on the co-ordinate axes.

5. Compute $\lim_{x \to 0} \left(\frac{\sin ax}{\sin bx} \right)$, $b \neq 0$, $a \neq b$

6. Compute $\lim_{x \to \pi/2} \left(\frac{\cos x}{x - \pi/2} \right)$

7. If $y = \frac{a - x}{a + x} (x \neq -a)$, find $\frac{dy}{dx}$

8. If $y = (\cot^{-1} x^3)^2$, find $\frac{dy}{dx}$

9. If the increase in the side of a square is 2%, then find the approximate percentage of increase in its area.

10. Find the value of C in Lagrange's mean value theorem for the function $f(x) = x^2 - 1$ on [2, 3].

SECTION – B

5 × 4 = 20

Short answer type questions :

(i) Attempt any **five** questions.

(ii) Each question carries **four** marks.

11. Find the locus of the third vertex of a right angled triangle, the ends of whose hypotenuse are (4, 0) and (0, 4).

12. When the axes are rotated through an angle $\frac{\pi}{6}$, find the transformed equation of $x^2 + 2\sqrt{3}\,xy - y^2 = 2a^2$.

13. Find the value of k, if the lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent.

14. Find the real constants a, b, so that the function f given by

$$f(x) = \begin{cases} \sin x \quad , \text{if } x \leq 0 \\ x^2 + a \ , \text{if } 0 < x < 1 \\ bx + 3 \ , \text{if } 1 \leq x \leq 3 \ , f(x) = -3 \text{ if } x > 3 \end{cases}$$

is continuous on R.

15. Find the derivative of $x \sin x$ from the first principle.

16. Show that at any point (x, y) on the curve $y = b\, e^{x/a}$, the length of the sub-tangent is a constant and the length of the subnormal is $\frac{y^2}{a}$.

17. A particle is moving along a line according to $S = f(t) = 4t^3 - 3t^2 + 5t - 1$ where S is measured in metres and t is measured in seconds. Find the velocity and acceleration at time t. At what time the acceleration is zero ?

SECTION – C

5 x 7 = 35

Long answer type questions :

(i) Attempt any **five** questions.

(ii) Each question carries **seven** marks.

18. Find the circumcenter of the triangle whose vertices are (1, 3), (–3, 5) and (5, –1).

19. If the equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines, then show that the angle θ between the lines is given by

$$\cos\theta = \frac{|a + b|}{\sqrt{(a - b)^2 + 4h^2}}$$

20. Show that the lines joining the origin to the points of intersection of the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ and the straight line $x - y - \sqrt{2} = 0$ are mutually perpendicular.

21. Find the angle between two diagonals of a cube.

22. If $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$ for $0 < |x| < 1$, find $\frac{dy}{dx}$.

23. Show that the equation of the tangent to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ $(a \neq 0, b \neq 0)$ at the point (a, b) is $\frac{x}{a} + \frac{y}{b} = 2$.

24. From a rectangular sheet of dimensions 30 cm × 80 cm four equal squares of side x cm are removed at the corners, and the sides are then turned up so as to form an open rectangular box. Find the value of x, so that the volume of the box is the greatest.