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Code No. 8187/E

FACULTY OF SCIENCE
B.Sc. V-Semester (CBCS) Examination, November / December 2019

Subject : Mathematics (Linear Algebra)

Paper – V (DSC)

Max. Marks: 60

Time : 3 Hours

PART – A (5 x 3 = 15 Marks)
(Short Answer Type)

Note : Answer any five of the following questions.

- 1 Define null space of a matrix.
- 2 Find a matrix A such that $w = \text{Col } A$,

$$\text{where } w = \begin{bmatrix} 6a-b \\ a+b \\ -7a \end{bmatrix} : a, b \in \mathbb{R}$$

- 3 Find the eigen values of the matrix

$$A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$$

- 4 Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- 5 If $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$, find a formula for A^k given that $A = PDP^{-1}$ where

$$P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}, D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$$

- 6 Define orthogonal set.
- 7 Define vector subspace with an example.
- 8 Find $[\text{dist}(u_1 - v)]^2$.

PART – B (3x15=45 Marks)
(Essay Answer Type)

Note: Answer ALL from the questions.

- 9 (a) Let $M_{2 \times 2}$ be the vector space of all 2×2 matrices and define $T: M_{2 \times 2} \rightarrow M_{2 \times 2}$ by $T(A) = A + A^T$ where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Show that T is a linear transformation.

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(b) Find the dimension of the subspace

$$H = \left\{ \begin{bmatrix} a-3b+6c \\ 5a+4d \\ b-2c-d \\ 5d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}$$

10 (a) Show that if $v_1, v_2, \dots, v_r$ are eigen vectors that correspond to distinct eigen value $\lambda_1, \lambda_2, \dots, \lambda_r$ of an $n \times n$ matrix A then the set $\{v_1, v_2, \dots, v_r\}$ is linearly independent.

OR

(b) Is $\lambda = 3$ an eigen value of $\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. If so find the one corresponding eigen vector.

11 (a) Diagonalize the matrix

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix} \text{ if possible}$$

OR

(b) Let $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Find the orthogonal projection of y on to u . The write y as the sum of two orthogonal vectors, one in $\text{span}\{u\}$ and other one orthogonal to u .
