Code No. 9007 / E

FACULTY OF SCIENCE

B.Sc. I-Semester (CBCS) Examination, November / December 2019

Subject : Mathematics (Differential and Integral Calculus)

Paper – I

Time : 3 Hours Max. Marks: 80

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)
Note : Answer any EIGHT of the following questions.

1 If f(x, y) = y cos xy then evaluate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$.

2 If $f(x, y) = \frac{xy}{x^2 + y^2}$ then evaluate $\frac{\partial^2 f}{\partial x \partial y}$.

3 If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

4 If H = f(y - z, z – x, x – y) then show that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$.

5 If $z = x^2 + y^2$, $x = at^2$, $y^2 = 2at$ them evaluate $\frac{dz}{dt}$.

6 Expand f(x, y) = x^2 + 2xy – ye as a Taylor's series in powers of (x – 1) and (y – 2).

7 Find the radius of curvature for the curve $y = \frac{30}{x}$ at P(3, 10).

8 Find the envelope of the family of curves y = m x + a m^3.

9 Using Newton's method, find the radius of curvature for the curve $x^3 + y^3 - 2x^2 + 6y = 0$ at the origin O(0, 0).

10 Find the length of the curve $y = x^{3/2}$ from x = 0 to x =4.

11 Find the length of the curve $x = e^{\theta}\sin\theta$, $y = {}^{\theta}\cos\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

12 Find the volume of the region generated by revolving the curve y = cosx , y = 0 form $x = 0$ to $x = \frac{\pi}{2}$ about x – axis.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)
Note: Answer ALL from the questions.

13 (a) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, $(x \neq y)$ then show that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$

..2

OR

(b) If $u(x,y)=\frac{y^2-x^2}{y^2+x^2}$ then using Euler's theorem show that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0$$

14 (a) If f(x, y) possesses continuous second order partial derivatives f_{xy} and f_{yx} then show that $f_{xy} = f_{yx}$

OR

(b) Show that the minimum value of

$$u(x,y)=xy+\frac{a^3}{x}+\frac{a^3}{y}$$ is $3a^2$.

15 (a) Find the evolute of the hyperbola $2xy = a^2$.

OR

(b) Find the envelope of the curve $\left(\frac{x}{a}\right)^m+\left(\frac{y}{b}\right)^m=1$ where $a^n + b^n = c^n$.

16 (a) Show that the length of the curve $x^2 = a^2(1-e^{y/a})$ measured form O(0, 0) to P(x, y) is $a\log\left(\frac{a+x}{a-x}\right)-x$.

OR

(b) Find the volume of the solid obtained by revolving one arc of the cycloid $x=a(\theta+\sin\theta),\ y=a(1+\cos\theta)$ about X – axis

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