Code No. 8073 / E

FACULTY OF SCIENCE

B.Sc. III-Semester (CBCS) Examination, November / December 2019

Subject : Mathematics (Real Analysis)
Paper – III (DSC)

Time : 3 Hours Max. Marks: 80

1209-18-474-021
5/12/19

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

Note : Answer any FIVE of the following questions.

1 Find $\lim_{n\to\infty} s_n$, where $s_n = \sqrt{n^2+1} - n$.

2 Prove that every convergent sequence is a Cauchy sequence.

3 Find the set of subsequential limits of the sequence $\{a_n\}$ where $a_n = \sin\left(\frac{n\pi}{3}\right)$.

4 Test the convergence of the series $\sum \frac{n}{n^2+3}$.

5 Find the interval of convergence of the series $\sum \frac{x^n}{n^2}$.

6 Define the uniform convergence of a sequence of functions.

7 If f is a bounded function on [a, b], prove that $L(f) \le U(f)$ under usual notations.

8 Prove that every continuous function f on [a, b] is integrable.

PART – B (4 x 15 = 60 Marks)
(Essay Answer Type)

Note: Answer ALL the following questions.

9 (a) Prove that :

(i) $\lim_{n\to\infty}\left(n^{\frac{1}{n}}\right) = 1$ (ii) $\lim_{n\to\infty}\left(a^{\frac{1}{n}}\right) = 1$ for a > 0.

OR

(b) Let (s_n) be a sequence in **R**. If $\lim s_n$ is defined (as a real number or $+\infty$ or $-\infty$), then prove that $\lim\sup s_n = \lim s_n = \lim\inf s_n$.

10 (a) If (s_n) converges to a positive real number s and (t_n) is any sequence then prove that $\lim\sup s_n t_n = s.\ \lim\sup t_n$.

OR

(b) State and prove the comparison test.

11 (a) Show that if the series Σg_n converges uniformly on a set S, then $\lim_{n\to\infty} \sup\{|g_n(x)| : x \in S\} = 0$.

OR

(b) Let $f_n(x) = n^2 x^n (1-x)$ *for* $x \in [0, 1]$. Then prove that the sequence does not converge uniformly on [0, 1].

12 (a) Prove that a bonaded function f on [a, b] is integrable if and only if for each $\epsilon > 0$ there exists a partition P of [a, b] such that $U(f, p) - L(f, p) < \epsilon$

OR

(b) If f is integralbe on [a, b], then prove that | f | is integrable on [a, b] and $\left|\int_a^b f\right| \le \int_a^b |f|$.

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