

FACULTIES OF ARTS AND SCIENCE**B.A. / B.Sc. I – Year Examination, March / April 2016****Subject : MATHEMATICS****Paper – I : Differential Equations and Solid Geometry****Time : 3 hours****Max. Marks : 100**

**Note : Answer Six questions from Part-A & Four questions from Part-B.
Choosing at least one from each Unit. Each question in Part-A carries
6 marks and in Part-B carries 16 marks.**

Part – A (6 X 6 = 36 Marks)**Unit - I**

- 1 Solve $\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$.
- 2 Find the orthogonal trajectories of the family of rectangular hyperbolas $y = c_1/x$.

Unit – II

- 3 Solve $y'' + 3y' + 2y = 12e^x$
- 4 Solve $(D^2 - 3D + 2)y = 3 \sin 2x$.

Unit - III

- 5 Find the equation of the plane which passes through the points $(-1, 1, 1)$, $(1, -1, 1)$ and $(1, 1, -1)$.
- 6 Find the point where the line joining $(2, -3, 1)$, $(3, -4, -5)$ cuts the plane $2x + y + z = 7$.

Unit - IV

- 7 Find the equation of the cone whose vertex is at the origin and the direction cosines of whose generators satisfy the relation $3l^2 - 4m^2 + 5n^2 = 0$.
- 8 Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 0$.

Part – B (4 X 16 = 64 Marks)**Unit - I**

- 9 a) Prove that the integrating factor of non-exact differential equation $Mdx + Ndy = 0$ is $1/Mx + Ny$ if the differential equation is homogeneous and $Mx + Ny \neq 0$.
b) Solve $(1 + y^2)dx = (\tan^{-1}y - x)dy$.
- 10 a) Explain the method of solving Clairaut's equation $y = px + f(p)$
b) Solve $(x^2 + y^2 + 2x)dx + 2ydy = 0$

Unit - II

11 a) Explain the method of solving second order Cauchy Euler equation

$a_2 x^2 \frac{d^2 y}{dx^2} + a_1 x \frac{dy}{dx} + a_0 y = Q(x)$ where a_0, a_1 and a_2 are constants which are non-zero.

b) Solve $(D^2 - 3D + 2)y = xe^{2x} + \sin x$.

12 a) Solve $(D^2 + 4D + 4)y = 4x^2 + 6e^x$ by undetermined coefficients.

b) Apply method of variation of parameters to solve $(D^2 - 2D)y = e^x \sin x$.

Unit - III

13 a) A variable plane is at a constant distance $3p$ from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the triangle ABC is $x^2 + y^2 + z^2 = p^2$

b) Find the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{3} \text{ and } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}.$$

14 a) Find the equation of the sphere which pass through the points $(0,0,0)$, $(0,1,-1)$, $(-1,2,0)$ and $(1,2,3)$.

b) Find the equation of the sphere which pass through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch the plane $4x + 3y = 15$.

Unit - IV

15 a) Prove that $2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y + 26z - 17 = 0$ represents a cone with vertex at $(2,2,1)$.

b) Find the angle between the lines of intersection of $4x - y - 5z = 0$ and $8yz + 3zx - 5xy = 0$.

16 a) Find the equation of the cylinder whose generators touch the sphere

$$x^2 + y^2 + z^2 = a^2 \text{ and are parallel to the line } \frac{x}{\ell} = \frac{y}{m} = \frac{z}{n}.$$

b) Find the equation of the right circular cylinder of radius 3 and whose axis is

$$\text{the line } \frac{x-1}{2} = \frac{y-3}{2} = \frac{5-z}{7}.$$
