Code No. 2019 / E

FACULTIES OF ARTS AND SCIENCE

B.A. / B.Sc. III – Year Examination, March / April 2016

Subject : MATHEMATICS

Paper – III : Linear Algebra and Vector Calculus

Time : 3 hours **Max. Marks : 100**

Note : Answer Six questions from Part-A & Four questions from Part-B. Choosing atleast one from each Unit. Each question in Part-A carries 6 marks and in Part-B carries 16 marks.

Part – A (6 X 6 = 36 Marks)

Unit - I

1 Prove that the linear span L(S) of any subset S of a vector space V(F) is a subspace of V.

2 If U(F) and V(F) are two vector spaces and T is a linear transformation from U into V, then prove that the null space N(T) of T is a subspace of U.

Unit - II

3 Find the eigen roots and the corresponding eigen vectors of the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

4 Prove that $S = \left\{\left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right), \left(\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}\right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right)\right\}$ is an orthonormal set in R^3 with standard inner product.

Unit - III

5 Evaluate $\int_C (2x^2 + y^2)dx + (3y - 4x)dy$ around the triangle ABC whose vertices are A(0, 0), B(2, 0) and C(2, 1).

6 Evaluate $\iint \frac{x-y}{x+y}\, dx\, dy$ over [0, 1 ; 0, 1].

Unit - IV

7 If $\vec{f} = yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k}$ then show that $\vec{i} \times \frac{\partial \vec{f}}{\partial x} + \vec{j} \times \frac{\partial \vec{f}}{\partial y} + \vec{k} \times \frac{\partial \vec{f}}{\partial z} = \vec{0}$

8 Show that $\int_S \left(ax\,\vec{i} + by\,\vec{j} + cz\,\vec{k}\right) \cdot \vec{N} ds = 4\frac{\pi}{3}(a+b+c)$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

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Part – B (4 X 16 = 64 Marks)

Unit - I

9 a) Prove that every non-empty subset of a Linearly Independent set of vectors is Linearly Independent.

b) Prove that every Linearly Independent subset of a finitely generated vector space V(F) is either a basis of V or can be extended to form a basis of V.

10 a) State and prove rank-nullity theorem.

b) Prove that zero transformation is a linear transformation.

Unit - II

11 a) Find characteristic values and characteristic vectors of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

b) Find the minimal polynomial for a real matrix. $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$.

12 a) Explain Gram-Schmidt orthonormalization process.

b) In an inner product space V(F), prove that $|(\alpha, \beta)| \le \|\alpha\| \cdot \|\beta\|$ for all $\alpha, \beta, \in V$

Unit - III

13 a) Prove that every continuous function is integrable.

b) Prove sufficient condition for the existence of the integral.

14 a) Change the order of integration and hence show that

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dxdy}{(1+e^y)\sqrt{1-x^2-y^2}} = \frac{\pi}{2}\log\left(\frac{2e}{1+e}\right)$$

b) Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2)\, dx\, dy$.

Unit - IV

15 a) If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\,\vec{j} + 20xz^2\,\vec{k}$ evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the straight line joining (0, 0, 0) to (1, 0, 0) end then from (1, 1, 0) to (1, 1, 1).

b) If $\vec{F} = (x + y^2)\vec{i} - 2xy\,\vec{j} + 2yz\,\vec{k}$. Evaluate $\int_S \vec{F}.\vec{N}\,ds$ where S is the surface of plane 2x+y+2z=6 in the first octant.

16 a) State and prove Green's theorem in a plane.

b) Evaluate $\oint_C (\cos x \sin y - xy)\,dx + \sin x \cos y\, dy$, by Green's theorem where C is the circle $x^2 + y^2 = 1$.

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