FACULTIES OF ARTS AND SCIENCE

B.A. / B.Sc. II – Year Examination, March / April 2016

Subject : MATHEMATICS

Paper – II : Abstract Algebra and Real Analysis

Time : 3 hours Max. Marks : 100

Note : Answer Six questions from Part-A & Four questions from Part-B. Choosing atleast one from each Unit. Each question in Part-A carries 6 marks and in Part-B carries 16 marks.

Part – A (6 X 6 = 36 Marks)

Unit - I

1 Prove that the set G = {0, 2, 4, 6} forms an abelian group with respect to addition modulo 8.

2 If R^* denotes group of non zero real numbers under multiplication. Then prove that $f : R^* \to R^*$ under multiplication given by $f(x)=|x|$ is a homomorphism and find its Kernal.

Unit - II

3 Define unit of a Ring R. Find all units of Z_{14}.

4 If $f(x) = 2x^3 + 4x^2 + 3x + 2$ and $g(x) = 3x^4 + 2x + 4$ are two polynomials in $\mathbb{Z}_5[x]$ then find their sum and product.

Unit - III

5 Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$.

6 Define Cauchy sequence. Prove that every Cauchy's sequence of real numbers is bounded.

Unit - IV

7 Using Lagrange's mean value theorem, prove that $\frac{x-1}{x} < \log x < x-1$ for $x > 1$

8 If $f(x)=x^2$ for $x \in [0, 4]$, calculate the Riemann sums with partition $P = \{0, 2, 3, 4\}$.

Part – B (4 X 16 = 64 Marks)

Unit - I

9 a) Show that a non empty subset H of a group G is a sub group of G if and only if $ab^{-1} \in H$ for all a, b $\in$ H.

b) Define a cyclic group. Prove that every cyclic group is abelian.

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10 a) State Lagranges theorem in groups. Prove that every group of prime order is cyclic.

b) If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$ and $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$ are two permutations in S_6. Then find $\sigma^{2014}\mu^{2015}$.

Unit - II

11 a) Define an Integral domain. Prove that every field is an integral domain.

b) Prove that a commutative ring with unity is a field if and only if it has only trivial ideals.

12 a) Define Ring homomorphism. Show that $\phi : C \to M_2(R)$ given by $\phi(a+ib) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ for all a, b $\in$ R, is an isomorphism of C into M_2 (R).

b) If f(x) $\in$ F[x] and f(x) is of degree 2 or 3, then prove that f(x) is irreducible over F if and only if it has no zeros in F.

Unit - III

13 a) Prove that every convergent sequence is bounded. Is converse true? Justify your answer.

b) Let $x_n = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots + \frac{1}{n^2}$ for each $n \in N$, then prove that (x_n) is increasing and bounded.

14 a) State and prove Root Test.

b) Test the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{\sqrt{n+1}-\sqrt{n}}{n}\right)$.

Unit - IV

15 a) State and prove Lagrange's mean value theorem.

b) Evaluate $\lim_{x \to 0}\left[\frac{1-\cos x}{x^2}\right]$.

16 a) State and prove fundamental theorem of integral calculus.

b) Define the i) partition of closed interval [a, b] ii) norm iii) Riemann sums and iv) Riemann integrals (lower and upper).

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