

GATE CE 2019 Set 1: Questions & Solutions**General Aptitude**

1.They have come a long way in _____ trust among the users.

- A. created
- B. creating
- C. creation
- D. create

Answer :B

Solution :

They have come a long way in "creating" trust among the users.

2.The CEO's decision to quit was as shocking to the Board as it was to _____.

- A. myself
- B. me
- C. I
- D. my

Answer :B

Solution :

"me"

3.The lecture was attended by quite _____ students, so the hall was not very _____.

- A. few, quite
- B. a few, quite
- C. few, quiet
- D. a few, quiet

Answer :D

Solution :

a few, quiet

4.If $E = 10$; $J = 20$; $O = 30$; and $T = 40$, what will be $P + E + S + T$?

- A. 82
- B. 164
- C. 120
- D. 51

Answer :C

Solution :

$$P = 16 \times 2 = 32$$

$$E = 5 \times 2 = 10$$

$$S = 19 \times 2 = 38$$

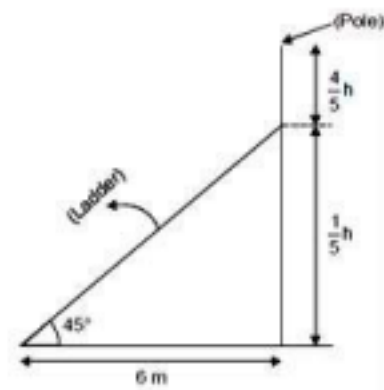
$$T = 20 \times 2 = 40$$

$$P + E + S + T = 120$$

5.On a horizontal ground, the base of straight ladder is 6m away from the base of a vertical pole. The ladder makes an angle of 45° to the horizontal. If the ladder is resting at a point located at one-fifth of the height of the pole from the bottom, the height of the pole is _____ meter.

Answer :

Solution :



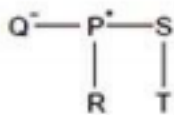
$$\begin{aligned} \tan 45^\circ &= \frac{h}{6} \\ \Rightarrow 1 &= \frac{h}{6} \\ \Rightarrow h &= 6 \text{ m} \end{aligned}$$

6. P, Q, R, S and T are related and belong to the same family. P is the brother of S. Q is the wife of P. R and T are the children of the siblings P and S respectively. Which one of the following statements is necessarily FALSE?

- A. S is the sister-in-law of Q
- B. S is the aunt of T
- C. S is the aunt of R
- D. S is the brother of P

Answer : B

Solution :



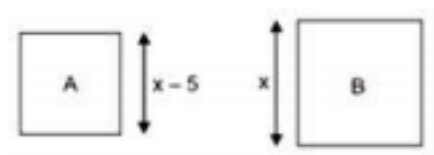
'T' is child of 'S'. So option (b) is right.

7. A square has sides 5cm smaller than the sides of a second square. The area of the larger square is four times the area of the smaller square. The side of the larger square is _____ cm.

- A. 15.10
- B. 18.50
- C. 10.00
- D. 8.50

Answer :C

Solution :



Given,

$$(\text{Area})_B = 4 \times (\text{Area})_A$$

$$\Rightarrow x^2 = 4(x-5)^2$$

$$\Rightarrow x^2 = 4[x^2 + 25 - 10x]$$

$$\Rightarrow x^2 = 4x^2 + 100 - 40x$$

$$\Rightarrow 3x^2 - 40x + 100 = 0$$

$$\Rightarrow 3x^2 - 30x - 10x + 100 = 0$$

$$\Rightarrow 3x(x-10) - 10(x-10) = 0$$

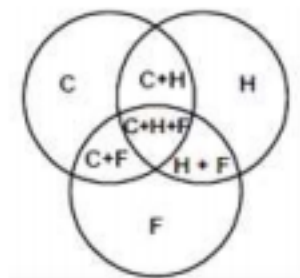
$$\Rightarrow x = 10 \text{ or } x = \frac{10}{3}$$

8. In a sports academy of 300 people, 105 play only cricket, 70 play only hockey, 50 play only football, 25 play both cricket and hockey, 15 play both hockey and football and 30 play both cricket and football. The rest of them play all three sports. What is percentage of people who play at least two sports?

- A. 23.30
- B. 50.00
- C. 28.00
- D. 25.00

Answer :D

Solution :



Total = 300

$C = 105$

$H = 70$

$F = 50$

$C + H = 25$

$H + F = 15$

$C + F = 30$

$C + H + F = 300 - (295) = 5$

% of people playing at least 25 sports

$$= \frac{25 + 15 + 30 + 5}{300} \times 100$$

$$= \frac{75}{300} \times 100 = 25\%$$

9. The increasing interest in tribal characters might be a mere coincidence, but the timing is of interest. None of this, though, is to say that the tribal hero has arrived in Hindi cinema, or that the new crop of characters represents the acceptance of the tribal character in the industry. The films and characters are too few to be described as a pattern.

Who does the word 'arrived' mean in the paragraph above?

- A. reached a terminus
- B. came to a conclusion
- C. attained a status

D. went to a place

Answer :C

Solution :

The word 'arrived' mean in the above para is to "Attained a status"

10.The new cotton technology, Bollgard-II, with herbicide tolerant traits has developed into a thriving business in India. However, the commercial use of this technology is not legal in India. Notwithstanding that, reports indicate that the herbicide tolerant Bt cotton had been purchased by farmers at an average of Rs 200 more than the control price of ordinary cotton, and planted in 15% of the cotton growing area in the 2017 Kharif season.

Which one of the following statements can be inferred from the given passage?

- A. Farmers want to access the new technology for experimental purposes
- B. Farmers want to access the new technology if India benefits from it
- C. Farmers want to access the new technology by paying high price
- D. Farmers want to access the new technology even if it is not legal

Answer :D

Solution :

Farmers want to access the new technology even if it is not legal

Civil Engineering

11.In a soil specimen, the total stress, effective stress, hydraulic gradient and critical hydraulic gradient are σ , σ' , i and i_c , respectively. For initiation of quicksand condition, which one of the following statement is TRUE?

- A. $\sigma' = 0$ and $i \neq i_c$
- B. $\sigma = 0$ and $i = i_c$
- C. $\sigma' \neq 0$ and $i \neq i_c$
- D. $\sigma' = 0$ and $i = i_c$

Answer :D

Solution :

During quick sand condition, the effective stress is reduced to zero [i.e, $\sigma' = 0$]

12. Assuming that there is no possibility of shear buckling in the web, the maximum reduction permitted by IS 800-2007 in the (low-shear) design bending strength of a semi-compact steel section due to high shear is

- A. 25%
- B. 50%
- C. governed by the area of the flange
- D. zero

Answer :D

Solution :

Maximum reduction permitted by IS 800-2007 is zero.

13. The coefficient of average rolling friction of a road is f_r and its grade is +G%. If the grade of this road is doubled, what will be the percentage change in the braking distance (for the design vehicle to come to stop) measured along the horizontal (assume all other parameters are kept unchanged)?

- A. $\frac{0.02G}{f_r + 0.01G} \times 100$
- B. $\frac{f_r}{f_r + 0.02G} \times 100$
- C. $\frac{0.01G}{f_r + 0.02G} \times 100$
- D. $\frac{2f_r}{f_r + 0.01G} \times 100$

Answer :C

Solution :

$$\text{Case I: Braking distance} = \frac{V^2}{2g(f_r + 0.01G)}$$

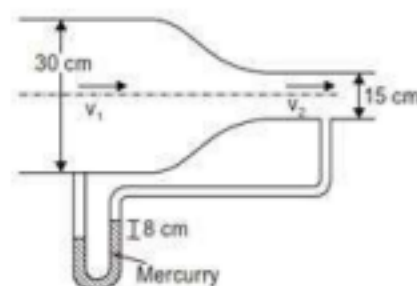
$$\text{Case II: Braking distance} = \frac{V^2}{2g(f_r + 0.02G)}$$

Percentage change

$$= \frac{\frac{V^2}{2g(f + 0.01G)} - \frac{V^2}{2g(f + 0.02G)}}{\frac{V^2}{2g(f + 0.01G)}} \times 100$$

$$= \frac{0.01G}{(f + 0.02G)} \times 100$$

14. A circular duct carrying water gradually contracts from a diameter of 30 cm to 15 cm. The figure (not drawn to scale) shows the arrangement of differential manometer attached to the duct.



When the water flows, the differential manometer shows a deflection of 8 cm of mercury (Hg). The values of specific gravity of mercury and water are 13.6 and 1.0, respectively. Consider the acceleration due to gravity, $g = 9.81 \text{ m/s}^2$. Assuming frictionless flow, the flow rate (in m^3/s , round off to 3 decimal places) through the duct is _____.

Answer :

Solution :

$$h = x \left(\frac{G_m}{G_w} - 1 \right)$$

$$h = 8 \left(\frac{13.6}{1} - 1 \right)$$

$$h = 100.8 \text{ cm} = 1.008 \text{ m}$$

Flow rate

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$$

$$A_1 = 4A_2, g = 9.81 \text{ m/sec}^2$$

$$A_2 = \frac{\pi}{4} \times 0.15^2$$

$$= 0.01767 \text{ m}^2$$

$$Q = \frac{4A_2^2}{\sqrt{16A_2^2 - A_1^2}} \sqrt{2 \times 9.81 \times 1.008}$$

$$= \frac{4A_2}{\sqrt{15}} \sqrt{19.777}$$

$$Q = \frac{4 \times (0.01767)}{\sqrt{15}} \sqrt{19.777}$$

$$Q = 0.081 \text{ m}^3/\text{sec}$$

15. A concentrated load of 500 kN is applied on an elastic half space. The ratio of the increase in vertical normal stress at depths of 2m and 4m along the point of the loading, as per Boussinesq's theory, would be _____.

Answer :

Solution :

Boussinesq's theory

$$= \frac{3Q}{2\pi z^2} \left(\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right)^{3/2}$$

$$r = 0$$

$$Q = \frac{3Q}{2\pi z^2}$$

$$Q \propto \frac{1}{z^2}$$

$$\frac{Q}{Q} = \left(\frac{4}{2}\right)^2 = 4$$

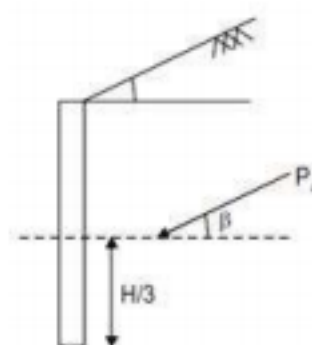
16. A retaining wall of height H with smooth vertical backface supports a backfill inclined at an angle β with the horizontal. The backfill consists of cohesionless soil having angle of internal friction ϕ . If the active lateral thrust acting on the wall is P_a which one of the following statements is TRUE?

- A. P_a acts at a height $H/3$ from the base of the wall and at an angle β with the horizontal
- B. P_a acts at a height $H/2$ from the base of the wall and at an angle ϕ with the horizontal

- C. P_a acts at a height $H/2$ from the base of the wall and at an angle β with the horizontal
 D. P_a acts at a height $H/3$ from the base of the wall and at an angle ϕ with the horizontal

Answer :A

Solution :



active thrust act at a height $H/3$ from the base of the wall and at an angle equal to backfill inclination.

17. In a rectangular channel, the ratio of the velocity head to the flow depth for critical flow condition, is

- A. $\frac{1}{2}$
 B. $\frac{2}{3}$
 C. $\frac{3}{2}$
 D. 2

Answer :A

Solution :

Velocity head for a critical flow

$$= \frac{q^2}{2gy_c^3} = \frac{y_c^3}{2y_c} = \frac{1}{2}y_c$$

So, ratio of velocity head to critical flow depth = $\frac{1}{2}$

18. The probability that the annual maximum flood discharge will exceed $25000 \text{ m}^3/\text{s}$, at least once in next 5 years is found to be 0. The return period of this flood event (in years, round off to 1 decimal place) is _____.

Answer :

Solution :

Probability exceed maximum discharge at least once in next 5 years is given by
 $= 1 - (1 - p)^n$

$$\Rightarrow 0.25 = 1 - (1 - P)^n \quad [n = 5 \text{ year}]$$

$$\Rightarrow P = 0.559$$

$$\Rightarrow \frac{1}{T} = 0.559$$

$$\Rightarrow T = 17.9 \text{ year}$$

19. The interior angles of four triangles are given below:

Triangle	Interior Angles
P	$85^\circ, 50^\circ, 45^\circ$
Q	$100^\circ, 55^\circ, 25^\circ$
R	$100^\circ, 45^\circ, 35^\circ$
S	$130^\circ, 30^\circ, 20^\circ$

Which of the triangles are ill-conditioned and should be avoided in Triangulation surveys?

- A. Both P and R
- B. Both Q and S
- C. Both P and S
- D. Both Q and R

Answer :B

Solution :

A triangle is said to be ill condition when angle is less than 30° and more than 120° .

So, triangle S is ill conditioned.

For well conditioned of triangulation two angle should not be almost equal.

So, only triangle Q or triangle R is ill-conditioned

So, ill-condition S and Q or S and R. option S and Q is given. So option (b) correct.

20. A catchment may be idealised as a rectangle. There are three rain gauges located inside the catchment at arbitrary locations. The average precipitation over the catchment is estimated by two methods: (i) Arithmetic mean (P_A) and (ii) Thiessen polygon (P_T). Which of the following statements is correct?

- A. P_A is always smaller than P_T
- B. There is no definite relationship between P_A and P_T
- C. P_A is always equal to P_T
- D. P_A is always greater than P_T

Answer :B

Solution :

There is no definite relationship between arithmetic mean and Thiessen polygon method.

Only it can be says that in Thiessen polygon method average value is more uniformly distributed as compared to arithmetic mean.

21. An isolated concrete pavement slab of length L is resting on a frictionless base. The temperature of the top and bottom fibre of the slab are T_t and T_b , respectively. Given: the coefficient of thermal expansion = α and the elastic modulus = E . Assuming $T_t > T_b$ and the unit weight of concrete as zero, the maximum thermal stress is calculated as

- A. zero
- B. $E\alpha(T_t - T_b)$
- C. $L\alpha(T_t - T_b)$
- D. $\frac{E\alpha(T_t - T_b)}{2}$

Answer :A

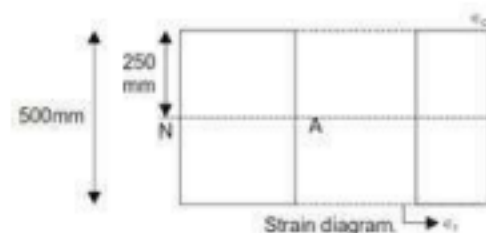
Solution :

Here thermal stress is zero.

22. For a given loading on a rectangular plain concrete beam with an overall depth of 500 mm, the compressive strain and tensile strain developed at the extreme fibers are of the same magnitude of 2.5×10^{-4} . The curvature in the beam cross-section (in m^{-1} , round off to 3 decimal places), is _____.

Answer :

Solution :



Given, $\epsilon = \epsilon_t = \epsilon_c = 2.5 \times 10^{-4}$

$y = 250 \text{ mm} = 0.25 \text{ m}$

As per flexure formula:

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\Rightarrow \frac{\sigma}{E \times y} = \frac{1}{R} \quad \text{Curvature of beam cross-section}$$

$$\Rightarrow \left(\frac{1}{R} \right) = \frac{\epsilon}{y} = \left(\frac{2.5 \times 10^{-4}}{0.25} \right) \text{m}^{-1}$$

$$= 0.001 \text{ m}^{-1}$$

23. For a small value of h , the Taylor series expansion of $f(x + h)$ is

- A. $f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{3} f'''(x) + \dots$
- B. $f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \dots$
- C. $f(x) - hf'(x) + \frac{h^2}{2!} f''(x) - \frac{h^3}{3!} f'''(x) + \dots$
- D. $f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{3} f'''(x) + \dots$

Answer :B

Solution :

For the small value of h, the Taylor's series expansion of

$$f(x+h) = \frac{f(x)}{0!} + \frac{hf'(x)}{1!} + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \dots$$

$$f(x+h) = f(x) + hf'(x) + \frac{h^2 f''(x)}{2!} + \frac{h^3}{3!} f'''(x) + \dots$$

So option (b) is correct

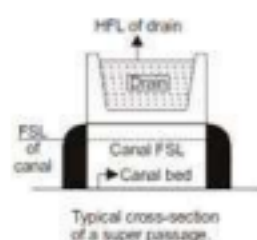
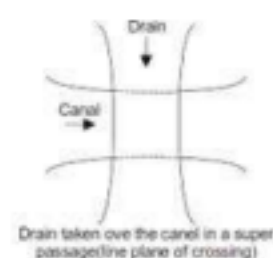
24.If the path of an irrigation canal is below the level of a natural stream, the type of cross drainage structure provided is

- A. Aqueduct
- B. Super passage
- C. Sluice gate
- D. Level crossing

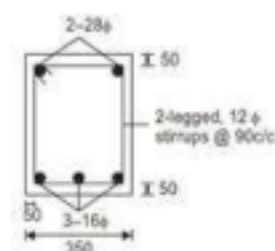
Answer :B

Solution :

If the path of an irrigation canal is below the bed level of a natural stream, the type of cross-drainage work provided is super passage.



25. In the reinforced beam section shown in the figure (not drawn to scale), the nominal cover provided at the bottom of the beam as per IS 456-2000, is

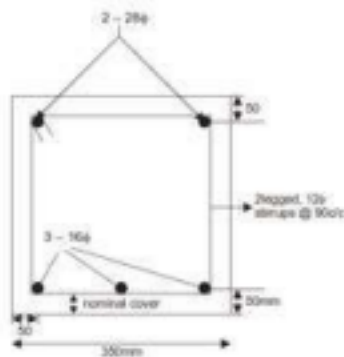


All dimensions are in mm

- A. 36 mm
- B. 50 mm
- C. 30 mm
- D. 42 mm

Answer :C

Solution :



$$\therefore \text{Nominal cover} = \left(50 - \frac{16}{2} - 12 \right) \text{mm}$$

$$= 30 \text{ mm}$$

26. Consider a two-dimensional flow through isotropic soil along x-direction and z-direction. If h is the hydraulic head, the Laplace's equation of continuity is expressed as

- A. $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial x \partial z} + \frac{\partial^2 h}{\partial z^2} = 0$
- B. $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$
- C. $\frac{\partial h}{\partial x} + \frac{\partial h}{\partial x \partial z} + \frac{\partial h}{\partial z} = 0$
- D. $\frac{\partial h}{\partial x} + \frac{\partial h}{\partial z} = 0$

Answer : B

Solution :

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

For homogeneous isotropic soils, the Laplace's equation of continuity is expressed as:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

27. A soil has specific gravity of its solids equal to 2. The mass density of water is 1000 kg/m^3 . Considering zero air voids and 10% moisture content of the soil sample, the dry density (in kg/m^3 , round off to 1 decimal place) would be _____.

Answer :

Solution :

$$G_s = 2.65$$

$$\rho_w = 1000 \text{ kg/m}^3$$

$$\eta_a = 0$$

$$w = 10\% = 0.10$$

$$= \frac{[(1 - 0) \times 2.65 \times 1000]}{1 + 0.1 \times 2.65}$$

$$= 2094.862 \text{ kg/m}^3$$

28. Which one of the following is correct?

A. $\lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sin 2x} \right) = 2$ and $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right) = 1$

B. $\lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sin 2x} \right) = \infty$ and $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right) = 1$

C. $\lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sin 2x} \right) = 1$ and $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right) = 1$

D. $\lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sin 2x} \right) = 2$ and $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right) = \infty$

Answer : A

Solution :

$$\lim_{x \rightarrow 0} \left(\frac{\sin 4x}{\sin 2x} \right) \left(\frac{0}{0} \text{ form} \right)$$

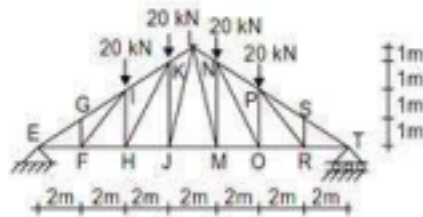
$$= \lim_{x \rightarrow 0} \frac{4 \cos 4x}{2 \sin 2x} = 2$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} \left(\frac{0}{0} \text{ form} \right)$$

$$\lim_{x \rightarrow 0} \frac{\sec^2 x}{1} = 1$$

So, option (a) is correct.

29. A plane truss is shown in the figure (not drawn to scale).



Which one of the options contains ONLY zero force members in the truss?

- A. FI, HI, PR, RS
- B. FI, FG, RS, PR
- C. FG, FI, HI, RS
- D. FG, FH, HI, RS

Answer :B

Solution :

Only 4 member having zero force member GF, FI and SR, PR

30. A simple mass-spring oscillatory system consists of a mass m , suspended from a spring of stiffness k . Considering z as the displacement of the system at any time t , the equation of motion for the free vibration of the system is $m\ddot{z} + kz = 0$. The natural frequency of the system is

- A. $\sqrt{\frac{k}{m}}$
- B. $\sqrt{\frac{m}{k}}$
- C. $\frac{k}{m}$
- D. $\frac{m}{k}$

Answer :A

Solution :

For simple harmonic motion

$$m\ddot{z} + kz = 0$$

$$\ddot{z} + \frac{k}{m}z = 0$$

Standard equation is $\frac{d^2x}{dt^2} + w^2x = 0$

$$\Rightarrow w^2 = \frac{k}{m}$$

$$\Rightarrow w = \sqrt{\frac{k}{m}}$$

31. A completely mixed dilute suspension of sand particles having diameters 0.25, 0.35, 0.40, 0.45 and 0.50 mm are filled in a transparent glass column of diameter 10 cm and height 2.50 m. The suspension is allowed to settle without any disturbance. It is observed that all particles of diameter 0.35 mm settle to the bottom of the column in 30 s. For the same period of 30 s, the percentage removal (round off to integer value) of particles of diameters 0.45 and 0.50 mm from the suspension is _____.

Answer :

Solution :

As we know that settling velocity for discrete particles is given by Stokes law as-

$$V_r = \frac{(G - 1) \gamma d^2}{18\mu}$$

$$\Rightarrow V_r \propto d^2$$

For 30 second duration if 0.35 mm particle size settles completely then % removal of particle size 0.45 mm and 0.50 mm will be 100% respectively for each. As settling velocity of particle size 0.45 mm and 0.50 mm will be greater than settling velocity of size 0.35 mm ($V_r \propto d^2$).

32. The maximum number of vehicles observed in any five minute period during the peak hour, If the total flow in the peak hour is 1000 vehicles, the five minute peak hour factor (round off to 2 decimal places) is _____.

Answer :

Solution :

Five minute peak hour factor

$$= \frac{\text{Average flow during 1 hour}}{12 \times \text{peak flow during 5 minute}}$$

$$PHF = \frac{V_{av}^{60}}{12 \times V_{5v}^5} = \frac{1000}{12 \times 160} = 0.52$$

33. Which one of the following is secondary pollutant?

- A. Carbon Monoxide
- B. Hydrocarbon
- C. Volatile Organic Carbon (VOC)
- D. Ozone

Answer :D

Solution :

Ozone is a secondary pollutant

34. An element is subjected to biaxial normal tensile strains of 0.0030 and 0.0020. The normal strain in the plane of maximum shear strain is

- A. 0.0050
- B. Zero
- C. 0.0025
- D. 0.0010

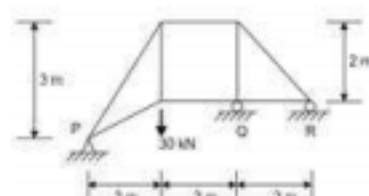
Answer :C

Solution :

$$(\epsilon)_{\text{in max shear strain}} = \frac{\epsilon_1 + \epsilon_2}{2}$$

$$= \frac{0.0030 + 0.0020}{2} = 0.0025$$

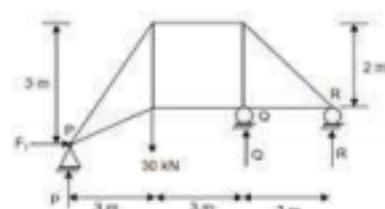
35. Consider the pin-jointed plane truss shown in the figure (not drawn to scale). Let R_P , R_Q , and R_R denote the vertical reactions (upward positive) applied by the supports at P, Q, and R, respectively, on the truss. The correct combination of (R_P, R_Q, R_R) is represented by



- A. (20, 0, 10) kN
- B. (10, 30, -10) kN
- C. (30, -30, 30) kN
- D. (0, 60, -30) kN

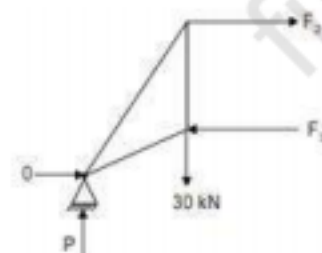
Answer :C

Solution :



$$\sum F_H = 0$$

$$F_1 = 0 \dots (i)$$



$$\sum F_V = 0$$

$$P = 30 \dots (ii)$$

$$\sum F_H = 0$$

$$F_2 = F_3 \dots (iii)$$

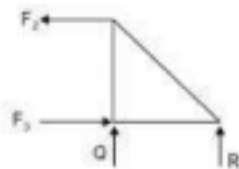
$$F_2 \times 3 + 30 \times 3 - F_3 \times 1 = 0$$

$$F_2 \times 3 - F_2 \times 1 + 90 = 0$$

$$2F_2 = -90$$

$$F_2 = -45$$

$$\Rightarrow F_3 = -45$$



$$Q + R = 0 \dots (iv)$$

$$F_3 \times 2 + R \times 3 = 0$$

$$-45 \times 2 + R \times 3 = 0$$

$$\boxed{R = 30} \Rightarrow \boxed{Q = -30}$$

36. For the following statements:

P – The lateral stress in the soil while being tested in an oedometer is always at-rest.

Q – For a perfectly rigid strip footing at deeper depths in a sand deposit, the vertical normal contact stress at the footing edge is greater than that at its centre.

R – The corrections for overburden pressure and dilatancy are not applied to measured SPTN values in case of clay deposits.

The correct combination of the statements is

- A. P – TRUE; Q – TRUE; R – FALSE
- B. P – TRUE; Q – TRUE; R – TRUE
- C. P – FALSE; Q – FALSE; R – TRUE
- D. P – FALSE; Q – FALSE; R – FALSE

Answer : B

Solution :

P – TRUE; Q – TRUE; R – TRUE

37. Tie bars of 12 mm diameter are to be provided in a concrete pavement slab. The working tensile stress of the tie bars is 230 MPa, the average bond strength between a tie bar and concrete is 2 MPa, and the joint gap between the slab is 10mm. Ignoring the loss of bond and the tolerance factor, the design length of the tie bars (in mm, round off to the nearest integer) is _____.

Answer :

Solution :

Given:

$$d = 12 \text{ mm}$$

$$\sigma_{st} = 230 \text{ MPa} = 230 \text{ N/mm}^2$$

$$S_b = 2 \text{ MPa} = 2 \text{ N/mm}^2$$

$$t = 10 \text{ mm}$$

$$\text{Length of tie bar} = t + \frac{d \sigma_{st}}{2 \times S_b}$$

$$= 10 + 690$$

$$= 700 \text{ mm}$$

38. Average free flow speed and the jam density observed on a road stretch are 60 km/h and 120 vehicles/km, respectively. For a linear speed-density relationship, the maximum flow on the road stretch (in vehicles/h) is _____.

Answer :

Solution :

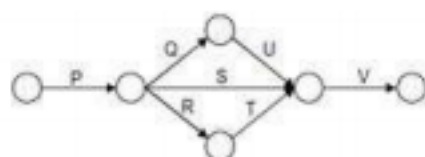
$$V_f = 60 \text{ km/h}$$

$$k_j = 120 \text{ Veh/km}$$

$$\therefore Q_{\text{min}} = \frac{V_k}{4}$$

$$= \frac{60 \times 120}{4} = 1800$$

39. The network of a small construction project awarded to a contractor is shown in the following figure. The normal duration, crash duration, normal cost, and crash cost of all the activities are shown in the table. The indirect cost incurred by the contractor is INR 5000 per day.

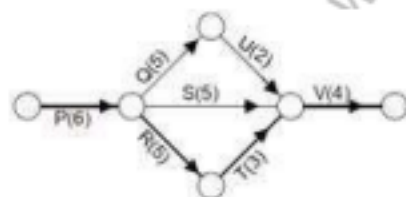


Activity	Normal Duration (days)	Crash Duration (days)	Normal Cost (NR)	Crash Cost (INR)
P	6	4	15000	25000
Q	5	2	6000	12000
R	5	3	8000	9500
S	6	3	7000	10000
T	3	2	6000	9000
U	2	1	4000	6000
V	4	2	20000	28000

If the project is targeted for completion in 16 days, the total cost (in INR) to be incurred by the contractor would be _____

Answer :

Solution :



Path PRTV is critical path and corresponding normal duration is 18 days.

Activity	t_n	t_c	C_n	C_c	$c_s = \frac{C_c - C_n}{t_n - t_c}$
P	6	4	15000	25000	5000
Q	5	2	6000	12000	2000
R	5	3	8000	9500	750
S	6	3	7000	10000	1000
T	3	2	6000	9000	3000
U	2	1	4000	6000	2000
V	1	2	2800	2800	4000

For 18 days:

Direct cost = 66000

Indirect cost = $18 \times 5000 = 90000$

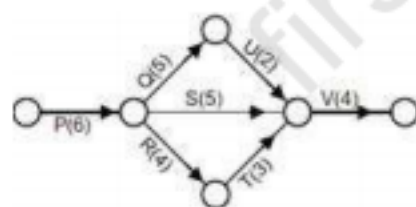
Total project cost = 156000

1st stage crashing :

Crash activity R by 1 day.

New project duration = 17 days.

T.P.C = $156000 + 1 \times 750 - 1 \times 5000 = 151750$



2nd stage crashing :

Crash activity Q & R simultaneously by 1 day. New project duration = 16 days.

T.P.C = $151750 + 1 \times (750 + 2000) - 5000 = 149500$

40. A sample of air analyzed at 0°C and 1 atm pressure is reported to contain 0.02 ppm (parts per million) of NO_2 . Assume the gram molecular mass of NO_2 as 46 and its volume at 0°C

and 1 atm pressure as 22.4 litres per mole. The equivalent NO_2 concentration (in microgram per cubic meter, round off to 2 decimal places) would be _____

Answer :

Solution :

$$0.02 \text{ ppm of } \text{NO}_2 \text{ means } = \frac{0.02 \text{ NO}_2}{10^6 \text{ l of air}}$$

$$= \frac{0.02}{22.4} \text{ mole NO}_2$$

$$= \frac{0.02}{22.4} \times 46 \text{ g}$$

$$= \frac{0.02}{22.4} \times \frac{46 \times 10^3}{10^6} \text{ g}$$

$$= \frac{0.02 \times 46}{22.4} \times \frac{10^3 \times 10^3}{10^6} \mu\text{g}$$

$$= 0.04107 \mu\text{g / l}$$

$$= 407 \mu\text{g/m}^3$$

41. Traffic on a highway is moving at a rate 360 vehicles per hour at a location. If the number of vehicles arriving on this highway follows Poisson distribution, the probability (round off to 2 decimal places) that the headway between successive vehicles lies between 6 and 10 seconds is _____

Answer :

Solution :

$$\lambda = 360 \text{ veh/hr}$$

$$= \frac{360 \text{ veh}}{3600 \text{ sec}} = 0.1 \text{ veh/sec}$$

$$P(6 \rightarrow 10) = \frac{(1t_2)^0 \times e^{-1t_2}}{0!} - \frac{(1t_1)^0 \times e^{-1t_1}}{0!}$$

$$= \frac{1 \times e^{-0.1 \times 10}}{1} - \frac{(0.1 \times 10)^0 \times e^{-0.1 \times 6}}{1}$$

$$= 0.18$$

42. Consider the ordinary differential equation $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$. Given the values of $y(1) = 0$ and $y(2) = 2$, the value of $y(3)$ (round off to 1 decimal place), is _____

Answer :

Solution :

$$x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

$$y(1) = 0 \text{ \& } y(2) = 2$$

Assume $x = e^t$

$$\Rightarrow \frac{dx}{dt} = e^t = x$$

$$\Rightarrow \frac{dx}{x} = dt$$

$$\text{Then } \frac{dy}{dx} = \frac{dy}{x dt}$$

$$\Rightarrow x \frac{dy}{dt} = \frac{dy}{dt} = Dy \quad \left(D = \frac{d}{dt} \right)$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} = D(D-1)y$$

Putting this into ordinary differential equation

$$D(D-1)y - 2Dy + 2y = 0$$

$$[D^2 - 3D + 2] = 0$$

Auxillary equation is

$$m^2 - 3m + 2 = 0$$

$$(m-1)(m-2) = 0$$

$$\text{i.e. } m = 1 \text{ \& } m = 2$$

\therefore solution of equation

$$y = c_1 e^t + c_2 e^{2t}$$

\Rightarrow Putting e^t as x

$$\Rightarrow y = c_1 x + c_2 x^2$$

$$\Rightarrow y(1) = 0$$

$$\Rightarrow c_1 + c_2 = 0 \dots(i)$$

$$\& y(2) = 2$$

$$\Rightarrow 2c_1 + 4c_2 = 2$$

$$\Rightarrow c_1 + 2c_2 = 1 \dots(ii)$$

$$(ii) - (i)$$

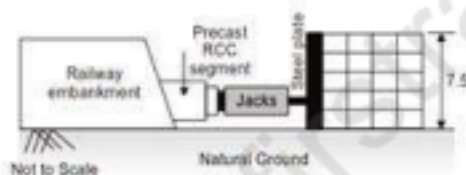
$$\Rightarrow c_2 = 1$$

$$c_1 = -1$$

$$\Rightarrow y = -x + x^2$$

$$\text{Then } y(3) = -3 + 3^2 = 6$$

43. A $3 \text{ m} \times 3 \text{ m}$ square precast reinforced concrete segments to be installed by pushing them through an existing railway embankment for making an underpass as shown in the figure. A reaction arrangement using precast PCC blocks placed on the ground is to be made for the jacks.



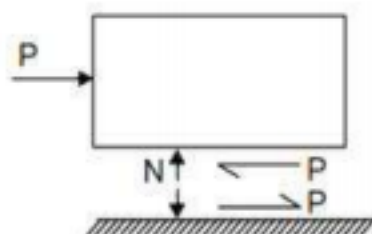
At each stage, the jacks are required to apply a force of 1875 kN to push the segment. The jacks will react against the rigid steel plate placed against the reaction arrangement. The footprint area of reaction arrangement on natural ground are: $c = 17 \text{ kPa}$; $\phi = 25^\circ$ and $\gamma = 18 \text{ kN/m}^3$. Assuming that the reaction arrangement has rough interface and has the same properties that of soil, the factor of safety (round off to 1 decimal place) against shear failure is _____.

Answer :

Solution :

FOS against shear failure

$$\frac{\text{Strength}}{\text{Applied load}} = \frac{(c + \sigma \tan \phi) A}{P}$$



$$\sigma = \frac{N}{A} = \frac{4 \frac{\text{kN}}{\text{m}^2} \times 37.5 \text{ m}^2 \times 7.5 \text{ m}}{37.5 \text{ m}^2}$$

$$= 24 \times 7.5 \text{ kN/m}^2$$

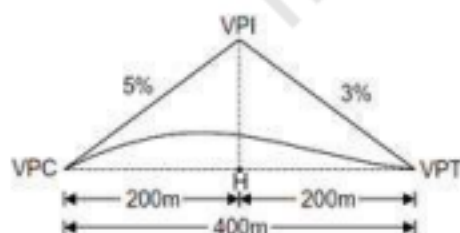
$$\Rightarrow \text{FOS} = \frac{(c + \sigma \tan \phi) A}{P} = \frac{(17 + 24 \times 7.5 \times \tan 25^\circ) \times 37.5}{1875}$$

$$\boxed{\text{FOS} = 2.0187}$$

44. A parabolic vertical curve is being designed to join a road of grade +5% with a road of grade -3%. The length of the vertical curve is 400 m measured along the horizontal. The vertical point of curvature (VPC) is located on the road of grade +5%. The difference in height between VPC and vertical point of intersection (VPI) (in m, round off to the nearest integer) is _____

Answer :

Solution :



Height difference between

$$\text{VPI \& VPC} = 5\% \text{ of } 200 \text{ m} = 10 \text{ m}$$

45. If the section shown in the figure turns from fully-elastic to fully-plastic, the depth of neutral axis (N.A.), decreases by

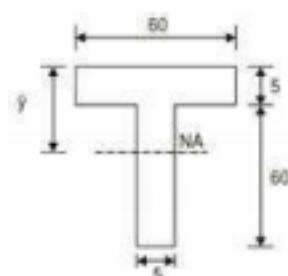


Figure not to scale
All dimensions are in mm

All dimensions are in mm

A.

13.75 mm

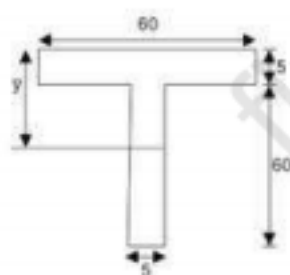
B. 10.75 mm

C. 15.25 mm

D. 12.25 mm

Answer : A

Solution :



For fully elastic case,

$$\bar{y} = \frac{60 \times 5 \times \frac{5}{2} + 60 \times 5 \times \left(5 + \frac{60}{2}\right)}{60 \times 5 + 60 \times 5}$$

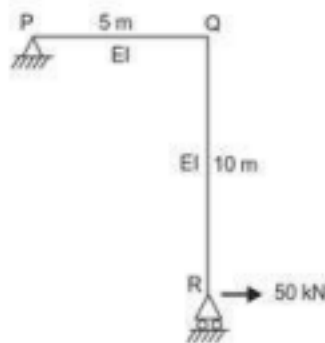
$$= \frac{750 + 10500}{2 \times 60 \times 5} = 18.75$$

For fully plastic case,

$y = \text{Equal area axis} = 5$

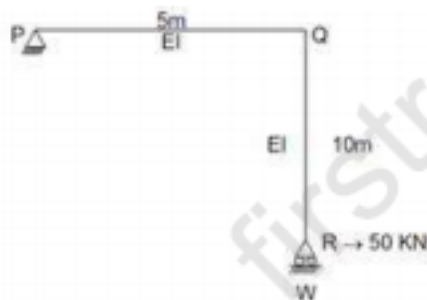
N.A reduces by $= 18.75 - 5 = 13.75 \text{ mm}$

46. A portal frame shown in figure (not drawn to scale) has a hinge support at joint P and a roller support at joint R. A point load of 50 kN is acting at joint R in the horizontal direction. The flexural rigidity, EI, of each member is 10^6 kNm^2 . Under the applied load, the horizontal displacement (in mm, round off to 1 decimal place) of joint R would be _____



Answer :

Solution :



For reaction

$$\sum M_P = 0$$

$$-W \times 5 + 50 \times 10 = 0$$

$$W = \frac{500}{5} = 100 \text{ kN}$$

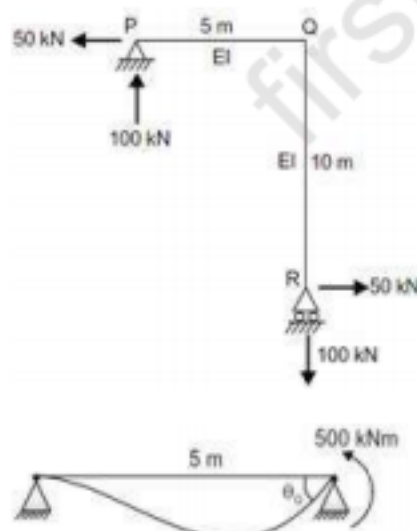
$$\therefore \delta = \int \frac{M \cdot m}{EI} dx$$

When unit load at R is acting in the direction of 50kN load, then reaction at R = 2 (downward)

Number	limit	M	m	EI
PQ	0-5m	$-100x+500$	$-2x+10$	10^6
QR	0-10 m	$50x$	x	10^6

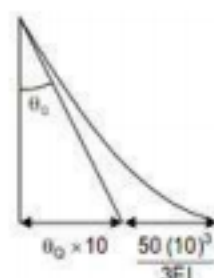
$$\begin{aligned}
 \delta &= \int_0^5 \frac{(-100x+500)(-2x+10)}{EI} dx \\
 &+ \int_0^{10} \frac{(50x)(x)}{EI} dx \\
 &= \int_0^5 \frac{(200x^2 - 1000x - 1000x + 5000)}{EI} dx \\
 &+ \int_0^{10} \frac{50x^2}{EI} dx \\
 &= \int_0^5 \frac{200x^2 - 2000x + 5000}{EI} dx + \int_0^{10} \frac{50x^2}{EI} dx \\
 &= \frac{1}{EI} \left[200 \times \frac{x^3}{3} - 2000 \frac{x^2}{2} + 5000x \right]_0^5 + \\
 &\quad \frac{1}{EI} \times 50 \left[\frac{x^3}{3} \right]_0^{10} \\
 &= \frac{1}{10^6} \left[\frac{200}{3} \times 125 - \frac{2000}{2} \times 25 + 5000 \times 5 \right] + \\
 &\quad \frac{50}{10^6} \times \frac{1}{3} \times 1000 = 25 \text{ mm}
 \end{aligned}$$

Alternatively:



$$\theta_0 = \frac{500 \times 5 \text{ kNm} - \text{m}}{3EI}$$

$$\theta_0 = \frac{2500}{3 \times 10^6} \text{ rad}$$



$$\Delta = \frac{10 \times 2500}{3 \times 10^6} + \frac{50(10)^3}{3 \times 10^6}$$

$$= \frac{75000}{3 \times 10^6} \text{ m} = 25 \times 10^{-3} \text{ m} = 25 \text{ mm}$$

47. A box measuring 50 cm × 50 cm × 50 cm is filled to the top with dry coarse aggregate of mass 187.5 kg. The water absorption and specific gravity of the aggregate are 0.5% and 2.5, respectively. The maximum quantity of water (in kg, round off to 2 decimal places) required to fill the box completely is _____

Answer :

Solution :

Volume of the box = 0.5 × 0.5 × 0.5

= 0.125 m³

Mass of aggregate = 187.5 kg

G_{agg} = 2.5

Volume of aggregate = $\frac{187.5}{2.5 \times 1000} = 0.075 \text{ m}^3$

Volume of empty space = 0.125 – 0.075

= 0.05 m³

Water absorption = 0.5%

Volume of water absorbed

$$= \frac{0.5}{100} \times \frac{187.5}{1000} = 9.375 \times 10^{-4}$$

Total volume of water that can be filled

$$= 9.375 \times 10^{-4} + 0.05$$

$$= 0.0509 \text{ m}^3$$

Mass of water = 50.94 kg

48. A wastewater is to be disinfected with 35mg/L of chlorine to obtain 99% kill of microorganisms. The number of micro-organisms remaining alive (N_t) at time t , is modelled by $N_t = N_0 e^{-kt}$, where N_0 is number of microorganisms at $t = 0$, and k is the rate of kill. The wastewater flow rate is $36 \text{ m}^3/\text{h}$, and $k = 0.23 \text{ min}^{-1}$. If the depth and width of the chlorination tank are 1.5 m and 1.0m, respectively, the length of the tank (in m, round off to 2 decimal places) is _____

Answer :

Solution :

For 99% kill of microorganism

$$\eta = \frac{N_0 - N_t}{N_0} = 0.99$$

$$N_t = 0.01 N_0$$

$$N_0 e^{-kt} = 0.01 N_0$$

$$-kt = \ln 0.01$$

$$-0.23t = -4.605$$

$$t = 20.022 \text{ min}$$

Volume of tank req. = $Q \cdot t$

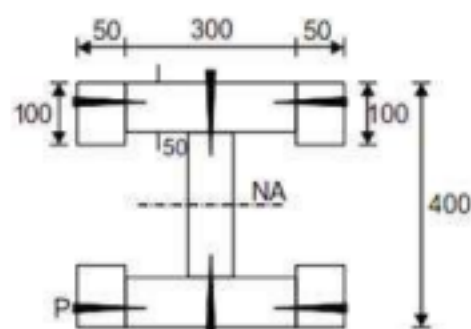
$$= 36 \frac{\text{m}^3}{\text{hr}} \times 20.02 \text{ min}$$

$$= \frac{36 \times 20.02}{60} \text{ m}^3$$

$$= 12.012 \text{ m}^3$$

$$\begin{aligned} \text{length} &= \frac{V}{\text{depth} \times \text{width}} \\ &= \frac{12.012}{1.5 \times 1} = 8.0089 \text{ m} \end{aligned}$$

49. The cross-section of a built-up wooden beam as shown in the figure (not drawn to scale) is subjected to a vertical shear force of 8 kN. The beam is symmetrical about the neutral axis (N.A.) shown, and the moment of inertia about N.A. is $1.5 \times 10^9 \text{ mm}^4$. Considering that the nails at the location P are spaced longitudinally (along the length of the beam) at 60 mm, each of the nails at P will be subjected to the shear force of



All dimensions are in mm

A.

60 N

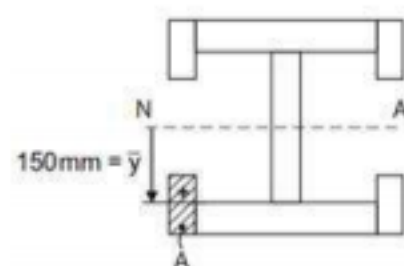
B. 120 N

C. 240 N

D. 480 N

Answer : C

Solution :



Shear force in nail at $P = \frac{VA\bar{y}}{I} \times \text{pitch}$

$$= \frac{8000 \times 100 \times 50 \times 150 \text{ Nmm}^3}{1.5 \times 10^8 \text{ mm}^4} \times 60 \text{ mm}$$

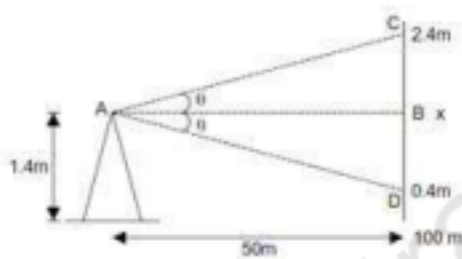
$$= 240 \text{ N}$$

So shear force in nail = $22 \times 60 = 1320 \text{ N}$

50. A staff is placed on a benchmark (BM) of reduced level (RL) 100.000 m and a theodolite is placed at a horizontal distance of 50m from the BM to measure the vertical angles. The measured vertical angles from the horizontal at the staff readings of 0.400m and 2.400 m are found to be the same. Taking the height of the instrument as 1.400 m, the RL (in m) of the theodolite station is _____

Answer :

Solution :



$$\tan \theta = \frac{2.4 - x}{50} = \frac{x - 0.4}{50}$$

$$2x = 2.8$$

$$x = 1.4\text{m}$$

$$\text{H.O.I} = 100 + 1.4$$

$$= 101.4\text{m}$$

$$\text{RL of theodolite station} = 101.4 - \text{theodolite height}$$

$$= 101.4 - 1.4$$

$$= 100\text{m}$$

51. A 0.80 m deep bed of sand filter (length 4m and width 3m) is made of uniform particles (diameter = 0.40 mm, specific gravity = 2.65, shape factor = 0.85) with bed porosity of 0.4. The bed has to be backwashed at a flow rate of 3.60 m³/min. During backwashing, if the terminal settling velocity of sand particles is 0.05 m/s, the expanded bed depth (in m, round off to 2 decimal places) is _____

Answer :

Solution :

$$n_{ex} = \left(\frac{V_b}{V_t} \right)^{0.22}$$

$$V_b = \frac{3.6}{4 \times 3 \times 60} = 5 \times 10^{-3} \text{ m/sec}$$

$$n_{ex} = \left(\frac{5 \times 10^{-3}}{0.05} \right)^{0.22}$$

$$\boxed{n_{ex} = 0.6025}$$

$$\text{then } L_{ex} (1 - n_{ex}) = L(1 - n)$$

$$\Rightarrow L_{ex} (1 - 0.6025) = 0.8 \times (1 - 0.4)$$

$$\boxed{L_{ex} = 1.2075 \text{ m}}$$

52. A reinforced concrete circular pile of 12m length and 0.6 m diameter is embedded in stiff clay which has an undrained unit cohesion of 110 kN/m². The adhesion factor is 0. The Net Ultimate Pullout (Uplift) Load for the pile (in kN, round off to 1 decimal place) is _____

Answer :

Solution :

$$\text{Pull out load} = \alpha C_u \cdot \ell \cdot p$$

p = perimeter

ℓ = length

$$= 0.5 \times 110 \times 12 \times \pi(0.6)$$

$$= 1244.07 \text{ kN}$$

53. A survey line was measured to be 285.5m with a tape having a nominal length of 30m. On checking, the true length of the tape was found to be 0.05 m too short. If the line lay on a slope of 1 in 10, the reduced length (horizontal length) of the line for plotting of survey work would be

- A. 285.0 m
- B. 284.5 m
- C. 285.6 m
- D. 283.6 m

Answer :D

Solution :

Measured length = 285.5 m

Nominal length of tape = 30 m

Slope = 1 in 10

The tape is 0.05 m too short

Actual length of tape = 30 – 0.05 = 29.95 m

Actual length measured

$$\begin{aligned}
 &= \frac{\text{Actual length of tape}}{\text{Nominal length of tape}} \times \text{Measured length} \\
 &= \frac{29.95}{30} \times 285.5 \\
 &= 285.024 \text{ m}
 \end{aligned}$$

Now slope correction = $-\frac{h^2}{2L}$

∵ Slope = 1 in 10

$$\begin{aligned}
 \Rightarrow h &= \frac{1}{10} \times 285.024 \\
 \Rightarrow h &= 28.5024 \text{ m}
 \end{aligned}$$

$$\Rightarrow \text{Slope correction} = \frac{-(28.5024)^2}{2 \times 285.024}$$

$$= -1.42512 \text{ m}$$

\Rightarrow Length to be plotted

= Actual length measured + correction

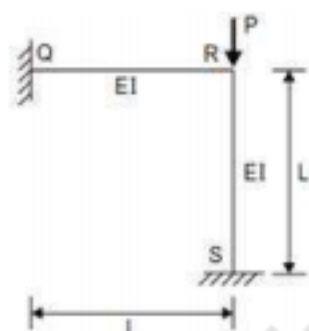
$$= 285.024 + (-1.42512)$$

$$= 283.599 \text{ m}$$

Hence, option (d) is correct.

54. The rigid-jointed plane frame QRS shown in the figure is subjected to a load P at the joint R. Let the axial deformation in the frame be neglected. If the support S undergoes a

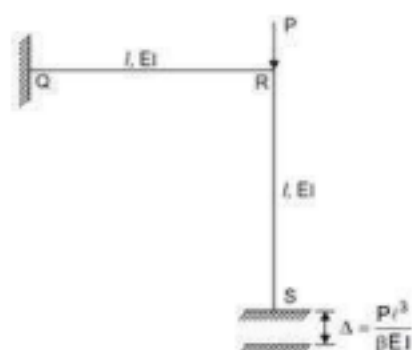
settlement of $\Delta = \frac{PL^2}{\beta EI}$, the vertical reaction at the support S will become zero when β is equal to



- A. 3.0
- B. 7.5
- C. 0.1
- D. 48.0

Answer : B

Solution :



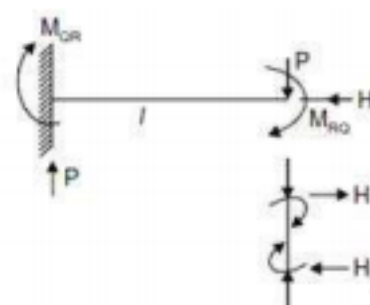
Using slope deflection method,

$$M_{QR} = \frac{2EI}{l} \left(\theta_R - \frac{3\Delta}{l} \right)$$

$$M_{RQ} = \frac{2EI}{l} \left(2\theta_R - \frac{3\Delta}{l} \right)$$

$$M_{RS} = \frac{2EI}{l} (2\theta_R)$$

If reaction at S is equal to zero



$$M_{RQ} + M_{QR} + Pl = 0$$

$$\frac{6EI\theta_R}{l} - \frac{12EI\Delta}{l^2} + Pl = 0$$

$$\frac{6EI\theta_R}{l} - \frac{12EI}{l^2} \times \frac{Pl^3}{8EI} + Pl = 0$$

$$\frac{6EI\theta_R}{l} - \frac{12Pl}{8} + Pl = 0 \quad \dots (i)$$

From equilibrium of joint

$$M_{RQ} + M_{RS} = 0$$

$$\frac{8EI\theta_R}{l} - \frac{6EI\Delta}{l^2} = 0$$

$$\frac{6EI\theta_R}{l} - \frac{6}{8} \left(\frac{6EI}{l^2} \times \frac{Pl^3}{8EI} \right)$$

$$\frac{6EI\theta_R}{l} = \frac{36Pl}{8} \quad \dots (ii)$$

⇒ From (i) & (ii)

$$\frac{36P_t}{8\beta} - \frac{96P_t}{8\beta} + P_t = 0$$

$$-\frac{60P_t}{8\beta} + P_t = 0$$

$$\Rightarrow 8\beta = 60$$

$$\beta = \frac{60}{8} = 7.5$$

55. Which one of the following is NOT a correct statement?

- A. The function $x\sqrt{x}$, ($x > 0$), has the global minima at $x = e$
- B. The function $x\sqrt{x}$, ($x > 0$), has the global maxima at $x = e$
- C. The function x^3 has neither global minima nor global maxima
- D. The function $|x|$ has the global minima at $x = 0$

Answer : A

Solution :

$$y = x^{1/x}$$

$$\ln y = \frac{1}{x} \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x} \left(\frac{1}{x} \right) + \ln x \left(\frac{-1}{x^2} \right)$$

$$\frac{dy}{dx} = x^{1/x} \times \frac{1}{x^2} (1 - \ln x)$$

$$\text{For } x > 0; \frac{dy}{dx} = 0$$

$$\Rightarrow x = e$$

Thus point $x = e$ is the critical point for $y = x^{1/x}$

Now at $x = e$, dy/dx changes its sign from (+ve) to (-ve). Thus point ($x = e$) is point of global maxima.

$y = x^3$ has neither global minima nor global maxima, it only have saddle point at $x = 0$

$y = |x|$; attains its minimum value at $x = 0$; so $x = 0$ is the global minima for $y = f(x)$

56. A rectangular open channel has a width of 5m and a bed slope of 0.001. For a uniform flow of depth 2m, the velocity is 2m/s. The Manning's roughness coefficient for the channel is

- A. 0.033
- B. 0.050
- C. 0.002
- D. 0.017

Answer :D

Solution :

For a rectangular channel

Width of channel = 5 m

Depth of flow = 2 m

Bed slope = 0.001

Velocity $V = 2$ m/sec.

From Manning's

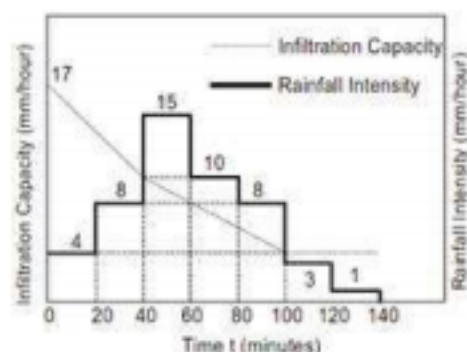
$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

$$\text{Where } R = \frac{A}{P} = \frac{5 \times 2}{5 + 2 \times 2} = 1.111 \text{ m}$$

$$\Rightarrow 2 = \frac{1}{n} \times (1.111)^{2/3} \times (0.001)^{1/2}$$

$$\Rightarrow n = 0.017$$

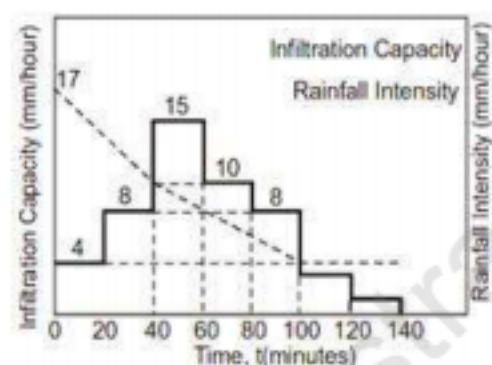
57. The hyetograph of a storm event of duration 140 minutes is shown in the figure.



The infiltration capacity at the start of this event ($t = 0$) is 17 mm/hour, which linearly decreases to 10 mm/hour after 40 minutes duration. As the event progresses, the infiltration rate further drops down linearly to attain a value of 4 mm/hour at $t = 100$ minutes and remains constant thereafter till the end of the storm event. The value of the infiltration index, ϕ (in mm/hour, round off to 2 decimal places), is _____

Answer :

Solution :

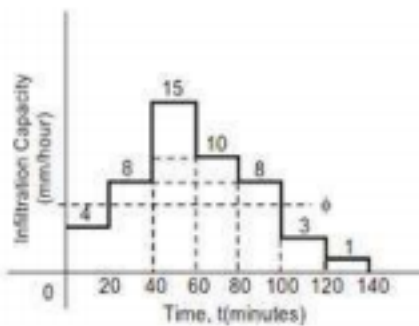


Depth of infiltration = Area of hyetograph above Horton's curve

$$= \left(15 \times \frac{20}{60} + 10 \times \frac{20}{60} + 8 \times \frac{20}{60} \right) - \frac{10 + 4}{2} \times \frac{60}{60}$$

$$= 4 \text{ mm}$$

Now, assuming $4 \leq \phi \leq 8$



$$(8 - \phi) \times \frac{20}{60} + (15 - \phi) \times \frac{20}{60} + (10 - \phi) \times \frac{20}{60}$$

$$(8 - \phi) \times \frac{20}{60} = 4$$

$$41 - 4\phi = 12$$

$$\Rightarrow \phi = 7.25 \text{ mm/hr}$$

58. Consider a laminar flow in the x-direction between two infinite parallel plates (Couette flow). The lower plate is stationary and the upper plate is moving with a velocity of 1 cm/s in the x-direction. The distance between the plates is 5 mm and the dynamic viscosity of the

fluid is 0.01 N-s/m². If the shear stress on the lower plate is zero, the pressure gradient, $\frac{\partial p}{\partial x}$ (in N/m² per m, round off to 1 decimal place) is _____

Answer :

Solution :

Given data ;

Velocity of plate, $V = 1 \text{ cm/sec}$

Distance between the late = 5 mm

Dynamic viscosity of fluid = 0.01 N-S/m²

Shear stress at lower plate = 0

Pressure gradient $\frac{\partial P}{\partial x} = ?$

We know that, in case of couette flow, shear stress (τ) is given by

$$\tau = \frac{\mu V}{B} + \left(-\frac{\partial P}{\partial x} \right) \left(\frac{B}{2} - y \right)$$

At lower plate, $y = 0$; $\tau = 0$ [Given]

$$0 = s \frac{0.01 \times 0.01}{0.005} - \left(\frac{\partial P}{\partial x} \right) \left[\frac{0.005}{2} - 0 \right]$$

$$\frac{\partial P}{\partial x} = 8 \text{ N/m}^2 \text{ per m}$$

59. A granular soil has a saturated unit weight of 20 kN/m^3 and an effective angle of shearing resistance of 30° . The unit weight of water is 9.81 kN/m^3 . A slope is to be made on this soil deposit in which the seepage occurs parallel to the slope up to the free surface. Under this seepage condition for a factor of safety of 1.5, the safe slope angle (in degree, round off to 1 decimal place) would be _____

Answer :

Solution :

$$\gamma_{\text{sat}} = 20 \text{ kN/m}^3$$

$$\phi = 30^\circ$$

$$\gamma_w = 9.81 \text{ kN/m}^3$$

$$\text{FOS} = 1.5$$

We know that

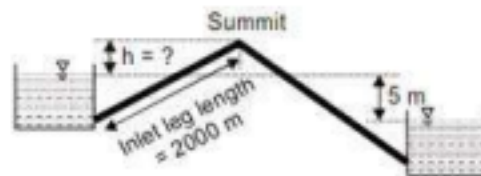
$$\text{FOS} = \frac{\gamma_{\text{sat}}}{\gamma_w} \times \frac{\tan \phi}{\tan i}$$

[i = safe slope angle]

$$1.5 = \frac{20 - 9.81}{20} \times \frac{\tan 30}{\tan i}$$

$$\Rightarrow i = 11.0953$$

60. Two water reservoirs are connected by a siphon (running full) of total length 5000 m and diameter of 0.10 m, as shown below (figure not drawn to scale).

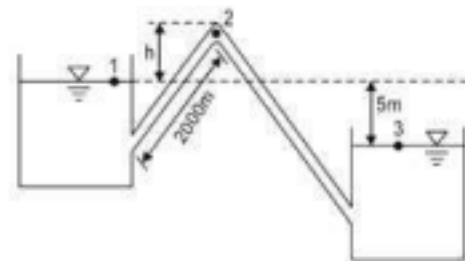


The inlet leg length of the siphon to its summit is 2000 m. The difference in the water surface levels of the two reservoirs is 5m. Assume the permissible minimum absolute pressure at the summit of siphon to be 2.5m of water when running full. Given: friction factor $f = 0.02$ throughout, atmospheric pressure = 10.3 m of water, and acceleration due to gravity $g = 9.81 \text{ m/s}^2$. Considering only major loss using Darcy- Weisbach equation the maximum height of the summit of siphon from the water level of upper reservoir, h (in m round off to 1 decimal place) is _____

Answer :

Solution :

Given data :



$$d = 0.1 \text{ m}$$

$$\text{Length of siphon} = 5000 \text{ m}$$

$$\text{Length of siphon upto summit} = 2000 \text{ m}$$

$$\text{Friction Factor, } f = 0.02$$

$$\text{Acceleration due to gravity, } g = 9.81 \text{ m/sec}^2$$

Applying Energy equation between point 1 and 3 to get

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 + h_{f(1-3)}$$

$$10.3 + 0 + Z_1 = 10.3 + 0 + Z_3 + \frac{fLQ^2}{12.1d^5}$$

$$\text{[From Darcy Weisbach equation]} \quad h_f = \frac{fLQ^2}{12.1d^5}$$

$$\Rightarrow 5 = \frac{0.02 \times 5000 \times Q^2}{12.1 \times (0.1)^5}$$

$$[\because Z_1 - Z_3 = 5\text{m}]$$

$$\Rightarrow Q = 2.4597 \times 10^{-3} \text{ m}^3/\text{sec}$$

Now applying energy equation between 1 and 2 to get

$$\begin{aligned} \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 &= \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_{f(1-2)} \\ 10.3 + 0 + Z_1 &= \frac{P_2}{\gamma} + \frac{Q^2}{2gA^2} + Z_2 + \frac{f_{(1-2)} Q^2}{12.1 d^5} \\ \Rightarrow 10.3 - (Z_2 - Z_1) &= 2.5 + \frac{(2.4597 \times 10^{-3})^2}{2 \times 9.81 \times \frac{\pi}{4} \times 0.1^2} \\ &+ \frac{0.02 \times 2000 \times (2.4597 \times 10^{-3})^2}{12.1 \times (0.1)^5} \\ \Rightarrow 10.3 - h &= 4.5 \text{ m} \\ \Rightarrow h &= 5.8 \text{ m} \end{aligned}$$

61. Sedimentation basin in a water treatment plant is designed for a flow rate of $0.2 \text{ m}^3/\text{s}$. The basin is rectangular with a length of 32m, width of 8m and depth of 4m. Assume that the settling velocity of these particles is governed by the Stokes' law. Given: density of the particles = 2.5 g/cm^3 ; density of water = 1 g/cm^3 ; dynamic viscosity of water = 0.01 g/(cm.s) ; gravitational acceleration = 980 cm/s^2 . If the incoming water contains particles of diameter $25 \mu\text{m}$ (spherical and uniform) the removal efficiency of these particles is

- A. 100%
- B. 65%
- C. 78%
- D. 51%

Answer : B

Solution :

Given :

Flow rate = $0.2 \text{ m}^3/\text{sec}$

Dimension of tank = $32\text{m} \times 8\text{m} \times 4\text{m}$

Density of particles = 2.5 g/cc

Density of water = 1 g/cc

Dynamic viscosity of water = 0.01 g/cm-S

Diameter of particle = $25\text{ }\mu\text{m}$

We know that

$$\text{Over flow rate of tank } (V_s) = \frac{0.2}{32 \times 8}$$

$$= 7.8125 \times 10^{-4} \text{ m/sec}$$

And settling velocity of particle (v_s),

$$v_s = \frac{(\gamma_s - \gamma_w) d^2}{18 \mu}$$

$$v_s = \frac{(2.5 - 1) \times 9.81 \times (25 \times 10^{-6})^2}{18 \times 0.01 \times 10^{-4}}$$

$$v_s = 5.1094 \times 10^{-4} \text{ m / sec}$$

$$\text{Now, \% removal efficiency} = \frac{v_s}{V_s} \times 100$$

$$= \frac{5.1094 \times 10^{-4}}{7.8125 \times 10^{-4}} \times 100$$

$$= 65.4\%$$

Hence option (b) is correct.

62. A square footing of 4m side is placed at 1 m depth in a sand deposit. The dry unit weight (γ) of sand is 15 kN/m^3 . This footing has an ultimate bearing capacity of 600 kPa . Consider the depth factors; $d_q = d_\gamma = 1.0$ and the bearing capacity factor: $N_\gamma = 18$. This footing is placed at a depth of 2m in the same soil deposit. For a factor of safety of 3.0 per Terzaghi's theory, the safe bearing capacity (in kPa) of this footing would be _____

Answer :

Solution :

Side of square footing = 4 m

Depth of footing = 1 m

Unit weight of soil = 15 KN/m³

Ultimate bearing capacity = 600 KPa

Depth factors, $d_q = d_\gamma = 1$

$N_\gamma = 18.75$

According to terzaghi, the ultimate bearing capacity of square footing is given as

At depth of footing = 1 m

$$q_u = 1.3C N_c + q N_q d_q + 0.4 B_\gamma N_\gamma d_\gamma$$

For sand, $C = 0$, $q = \gamma D_f = 15 \times 1 = 15 \text{ KN m}^2$

$$600 = 0 + 15 \times N_q \times 1 + 0.4 \times 4 \times 15 \times 18.75 \times 1$$

$$\Rightarrow N_q = 10$$

Now at depth of footing at 2m

$$q_u = 1.3 C N_c + q N_q + 0.4 B_\gamma N_\gamma d_\gamma$$

$$q_u = 0 + (2 \times 15)10 \times 1 + 0.4 \times 4 \times 15 \times 18.75 \times 1$$

$$q_u = 750 \text{ KPa}$$

\therefore We know that

$$q_{su} = q_u - \gamma D_f$$

$$q_{su} = 750 - 15 \times 2$$

$$q_{su} = 720 \text{ KPa}$$

and safe bearing capacity q_{safe}

$$\begin{aligned} q_{safe} &= \frac{q_{su}}{\text{FOS}} + \gamma D_f \\ &= \frac{720}{3} + 15 \times 2 \\ &= 270 \text{ KPa} \end{aligned}$$

63. Consider two functions $x = \Psi \ln \phi$ and $y = \phi \ln \Psi$. Which one of the following is the correct expression for $\frac{\partial \Psi}{\partial x}$?

- A. $\frac{x \ln \phi}{\ln \phi \ln \Psi - 1}$
 B. $\frac{\ln \phi}{\ln \phi \ln \Psi - 1}$
 C. $\frac{\ln \Psi}{\ln \phi \ln \Psi - 1}$
 D. $\frac{x \ln \Psi}{\ln \phi \ln \Psi - 1}$

Answer : C

Solution :

$$x = \Psi \ln \phi \Rightarrow \Psi = \frac{x}{\ln \phi} \quad \dots(i)$$

$$y = \phi \ln \Psi \Rightarrow \phi = \frac{y}{\ln \Psi}$$

Putting value of ϕ in (i)

$$\Psi = \frac{x}{\ln\left(\frac{y}{\ln \Psi}\right)} = \frac{x}{\ln y - \ln(\ln \Psi)} \quad \dots(ii)$$

Assuming y constant and differentiating Ψ w.r.t. x .

$$\frac{\partial \Psi}{\partial x} = \frac{(\ln y - \ln(\ln \Psi)) \cdot 1 - x \left(0 - \frac{1}{\ln \Psi} \cdot \frac{1}{\Psi} \cdot \frac{\partial \Psi}{\partial x}\right)}{(\ln y - \ln(\ln \Psi))^2} \quad \dots(iii)$$

Putting value of $(\ln y - \ln(\ln \Psi)) = \frac{x}{\Psi}$ from (ii) in equation (iii)

$$\frac{\partial \Psi}{\partial x} = \frac{\frac{x}{\Psi} + x \cdot \frac{1}{\Psi \ln \Psi} \cdot \frac{\partial \Psi}{\partial x}}{\left(\frac{x}{\Psi}\right)^2}$$

$$\Rightarrow \frac{\partial \Psi}{\partial x} = \frac{1 + \frac{1}{\Psi \ln \Psi} \frac{\partial \Psi}{\partial x}}{\left(\frac{x}{\Psi}\right)}$$

$$\Rightarrow \frac{x}{\Psi} \frac{\partial \Psi}{\partial x} = 1 + \frac{1}{\ln \Psi} \frac{\partial \Psi}{\partial x}$$

$$\Rightarrow \frac{\partial \Psi}{\partial x} \left(\frac{x}{\Psi} - \frac{1}{\ln \Psi} \right) = 1$$

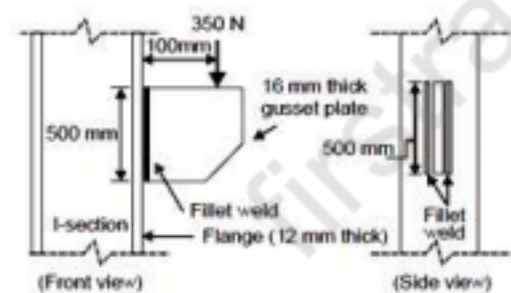
$$\Rightarrow \frac{\partial \Psi}{\partial x} = \frac{1}{\frac{x}{\Psi} - \frac{1}{\ln \Psi}} = \frac{\Psi \ln \Psi}{x \ln \Psi - \Psi}$$

$$\Rightarrow \frac{\partial \Psi}{\partial x} = \frac{\Psi \ln \Psi}{\Psi \ln \phi \ln \Psi - \Psi}$$

(replacing x by Ψ in ϕ)

$$\Rightarrow \frac{\partial \Psi}{\partial x} = \frac{\ln \Psi}{\ln \phi \ln \Psi - 1}$$

64. A 16 mm thick gusset plate is connected to the 12 mm thick flange plate of an I-section using fillet welds on both sides as shown in the figure (not drawn to scale). The gusset plate is subjected to a point load of 350 kN acting at a distance of 100 mm from the flange plate. Size of fillet weld is 10 mm.



The maximum resultant stress (in MPa, round off to 1 decimal place) on the fillet weld along the vertical plane would be _____

Answer :

Solution :

Given Data :

Thickness of gusset plate (t) = 16 mm

Point load (P) = 350 kN

Eccentricity (e) = 100 mm

Direct shear stress, $q = \frac{P}{A}$

$$q = \frac{350 \times 10^3}{2 \times 500 \times 10 \times 0.7}$$

$$q = 50 \text{ N/mm}^2$$

And bending stress on the extreme edge of weld (f)

$$f = \frac{M}{Z} = \frac{3Pe}{th^2}$$

$$f = \frac{3 \times 350 \times 10^3 \times 100}{0.7 \times 10 \times 500^2}$$

$$\Rightarrow f = 60 \text{ N/mm}^2$$

For checking the safety

Resultant Stress, $\sigma = \sqrt{f^2 + q^2}$

$$= 105.36 \text{ N/mm}^2$$

65. A one-dimensional domain is discretized into N sub-domains of width Δx with node numbers $i = 0, 1, 2, 3, \dots, N$. If the time scale is discretized in steps of Δt , the forward-time and centered-space finite difference approximation at n^{th} node and n^{th} time step, for the partial

differential equation $\frac{\partial v}{\partial t} = \beta \frac{\partial^2 v}{\partial x^2}$ is

- A. $\frac{v_i^{(n)} - v_i^{(n-1)}}{2\Delta t} = \beta \left[\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{2\Delta x} \right]$
- B. $\frac{v_i^{(n)} - v_i^{(n-1)}}{\Delta t} = \beta \left[\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{(\Delta x)^2} \right]$
- C. $\frac{v_{i+1}^{(n)} - v_i^{(n)}}{\Delta t} = \beta \left[\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{2\Delta x} \right]$
- D. $\frac{v_i^{(n+1)} - v_i^{(n)}}{\Delta t} = \beta \left[\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{(\Delta x)^2} \right]$

Answer : D

Solution :

Given differential equation

$$\frac{\partial v}{\partial t} = \beta \frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial v}{\partial t} = \frac{v_i^{(n+1)} - v_i^{(n)}}{(\Delta t)} \dots (i)$$

Using forward time finite difference

$$\frac{\partial^2 v}{\partial x^2} = \frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{(\Delta x)^2} \dots (ii)$$

Using centred space finite difference

$$\frac{\partial^2 v}{\partial x^2} = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Putting (i) and (ii) in PDE

$$\frac{v_i^{(n+1)} - v_i^{(n)}}{(\Delta t)} = \beta \left(\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{\Delta x^2} \right)$$

So, option (d) is correct.