

GATE_SOLUTION

GA

1. The strategies that the company uses to sell its products include house to house marketing.
2. The boat arrived at dawn
3. As the positions of book R & S are fixed. The books P, Q and T can be arranged in $3! = 6$ ways
4. When he did not come home, she pictured him lying dead on the roadside somewhere.
5. Let t be the time taken by the machines when they work simultaneously.

$$\therefore \frac{1}{t} = \frac{1}{4} + \frac{1}{2}$$

$$\therefore \frac{1}{t} = \frac{3}{4}$$

$$\therefore t = \frac{4}{3}$$

6. Given is the % of illiterates

So % of literates will be

	F	M
2001	40%	50%
2011	60%	60%

And population distribution is

	F	M
2001	40%	60%
2011	50%	50%

Let total population in both the years as T .

So total literate in 2001 will be

$$0.4 \times 0.4 + 0.5 \times 0.6 = 0.46T$$

And total literate in 2011 will be

$$0.5 \times 0.6 + 0.5 \times 0.6 = 0.6T$$

$$\therefore \text{Increase} = 0.6T - 0.46T = 0.14T$$

$$\therefore \% \text{ increase} = \frac{0.14T}{0.46T} \times 100 = 30.43$$

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8. As first line says Indian history was written by British historians was extremely well documented and researched, but not always impartial.

So option (C) can be interfered from given passage.

9.	P	Q
Start time	8 AM	8 AM
Working	$\frac{210}{360} \times 12 = 7 \text{ hrs}$	$\frac{210}{360} \times 12 = 8 \text{ hrs}$
Breaks	15 minutes each (2 breaks) = 30 minutes	20 minute break (1 break) = 20 minutes

$$\therefore \text{paid working hours} = 7 \text{ hrs} + 8 \text{ hrs} - 30 \text{ minutes} - 20 \text{ minutes}$$

$$= 14 \text{ hrs } 10 \text{ minutes}$$

$$\therefore \text{Paid} = 14 \times 200 + \frac{10}{60} \times 200$$

$$\therefore \text{Paid} = 2833.33$$

$$\therefore \text{Budget left} = 3000 - 2833.33 = 166.67$$

10. As it is given that R is sharing an office with T. So only option (D) is correct.

Electronics Engineering

1. A function $F(z)$ is said to be analytic at a point $z = a$ then $F(z)$ has a derivative at $z = a$ and derivative exists at each neighbouring point of $z = a$ in domain D .

$$\frac{1}{e^z} \text{ at } z = 0 \longrightarrow e^\infty \longrightarrow \text{No derivative}$$

$$\ln z \text{ at } z = 0 \rightarrow \ln(0) = -\infty \rightarrow \text{does not exists}$$

$$\frac{1}{1-z} \text{ at } z = 1 \rightarrow \frac{1}{0} = \infty \rightarrow \text{does not exists}$$

But $\cos z$ exists for all values of z so it is analytic over the entire complex plane.

2. As no supply is connected hence fermi level will be constant.

In P type semiconductor Fermi level should be closer to EV.

In N type semiconductor Fermi level should be closer to EC.

In P^{++} type semiconductor due to large doping Fermi level enters into valance band.

Hence answer is (B).

3. By reciprocity theorem,

$$\frac{1}{5} = \frac{I}{5}$$

$$\therefore I = 1A$$

4. let output of NAND gate is M and output of NOR gate is N

$$\therefore M = \overline{E_N \cdot D}$$

$$\text{And } N = \overline{\overline{E_N} + D}$$

$$\therefore N = E_N \cdot \overline{D}$$

When $E_N = 0$

$M = 1$ and $N = 0$

So both PMOS and NMOS will be OFF

So F will be at high impedance

When $E_N = 1$

$$M = \overline{D} \text{ \& } N = \overline{D}$$

So this CMOS will act as not gate

\therefore F will be D

\therefore Option (A) is correct.

5. Since it is a upper triangular matrix eigen values will be 2, 1, 3, 2

\therefore distinct eigen values are three

$$6. \quad \frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$$

When $n = -1$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\therefore \frac{dy}{y} = -\frac{dx}{x}$$

$$\therefore \ln y = -\ln(x) + \ln(c)$$

$$\therefore \ln(xy) = \ln(c)$$

$$\therefore xy = c$$

This represents rectangular hyperbola.

Now for $n = +1$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\therefore ydy = -x dx$$

$$\therefore \frac{y^2}{2} = \frac{-x^2}{2} + c$$

$$\therefore x^2 + y^2 = 2c$$

This represents family of circles.

7. let $H(z) = \frac{(z-a)(z-b)}{(z-c)(z-d)}$

$$\therefore H\left(\frac{1}{z}\right) = \frac{\left(\frac{1}{z}-a\right)\left(\frac{1}{z}-b\right)}{\left(\frac{1}{z}-c\right)\left(\frac{1}{z}-d\right)}$$

$$\therefore H\left(\frac{1}{z}\right) = \frac{\left(z-\frac{1}{a}\right)\left(z-\frac{1}{b}\right)}{\left(z-\frac{1}{c}\right)\left(z-\frac{1}{d}\right)}$$

$$\therefore H(z) \cdot H\left(\frac{1}{z}\right) = \frac{(z-a)(z-b)\left(z-\frac{1}{a}\right)\left(z-\frac{1}{b}\right)}{(z-c)(z-d)\left(z-\frac{1}{c}\right)\left(z-\frac{1}{d}\right)}$$

$$\therefore \text{zeros are } a, b, \frac{1}{a}, \frac{1}{b}$$

$$\text{given zero is } a = \frac{1}{2} + \frac{1}{2}j$$

as $h(n)$ is real valued signal another zero must be complex conjugate of this

$$\therefore b = \frac{1}{2} - \frac{1}{2}j$$

$$\text{Now } z_3 = \frac{1}{a} = \frac{1}{\frac{1}{2} + \frac{1}{2}j}$$

$$= \frac{2}{1+j}$$

$$= \frac{2(1-j)}{2}$$

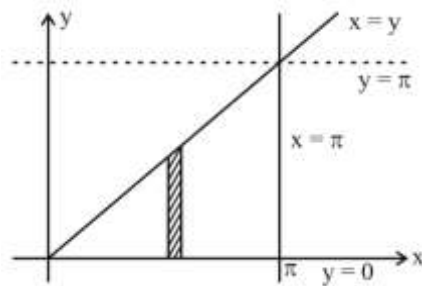
$$z_3 = 1 - j$$

as $h(n)$ is real valued signal another zero must be complex conjugate of this

$$z_4 = 1 + j$$

$$\therefore z_1 = \frac{1}{2} + \frac{1}{2}j \quad z_2 = \frac{1}{2} - \frac{1}{2}j \quad z_3 = 1 - j \quad z_4 = 1 + j$$

8.



By changing order of integration

$$\int_{x=0}^{x=\pi} \left(\int_{y=0}^{y=x} dy \right) \frac{\sin x}{x} dx$$

$$\therefore \int_{x=0}^{\pi} x \frac{\sin x}{x} dx$$

$$\therefore \int_{x=0}^{\pi} x \sin x dx$$

$$\therefore [-\cos x]_0^{\pi} = 2$$

$$9. R_{\text{rad}} = 80\pi^2 \left(\frac{dl}{\lambda} \right)^2$$

$$80\pi^2 \left(\frac{dlf}{c} \right)^2$$

$$\therefore R_{\text{rad}} \propto I^2 f^2$$

Now frequency is constant

$$\therefore R_{\text{rad}} \propto f^2$$

$$\therefore \frac{\Delta R}{R} = 2 \frac{\Delta f}{f}$$

$$= 2 \times 1\%$$

$$\therefore \frac{\Delta R}{R} = 2\%$$

10. $y(s)$ is unit step response

$$\therefore y(s) = G(s) \times \frac{1}{s}$$

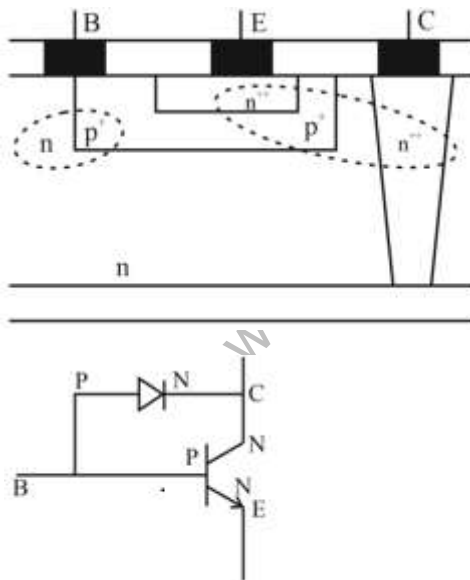
$$= \frac{3-s}{s(s+1)(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

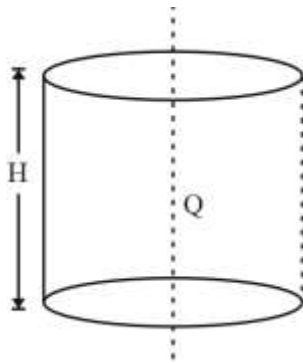
$$\therefore y(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+3}$$

$$\therefore y(t) = u(t) - 2e^{-t} u(t) + e^{-3t} u(t)$$

11.



12.



If we consider a total cylinder then by gauss law

○ used

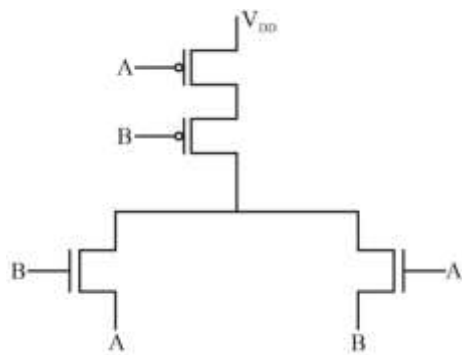
But $Q_{\text{enclosed}} = Q \cdot H$

And we are considering only $\frac{1}{4}$ th of the cylinder

$$\therefore D = \frac{Q \cdot H}{4}$$

$$\therefore E = \frac{Q \cdot H}{4 \epsilon_0}$$

13. By rearranging the circuit,



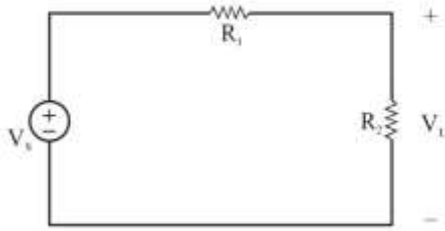
Truth table:

A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

So it is XNOR gate.

14. When V_s is +ve

Diode will be reverse biased



$$V_L = \frac{R_2}{R_1 + R_2} V_s$$

$$\therefore V_L = \frac{50}{50 + 50} \times 8$$

$$\therefore V_L = 4V \quad \dots(i)$$

When V_s is -ve

Diode will be forward biased



$$\therefore V_L = V_s = -10V \quad \dots(ii)$$

From (i) and (ii)

$$\text{Average value} = \frac{4 + (-10)}{2} = -3$$

$$\therefore \text{Average value} = -3$$

15. We know that

$$E[AX + BY] = AE[X] + BE[Y]$$

$$\therefore E[2X + Y] = 2E[X] + E[Y] = 0 \quad \dots(i)$$

$$\text{And } E[X + 2Y] = E[X] + 2E[Y] = 33 \quad \dots(ii)$$

Adding (i) and (ii)

$$3E[X] + 3E[Y] = 33$$

$$\therefore E[X] + E[Y] = 11$$

16. We know that

$$NM_L = V_{IL} - V_{OL}$$

$$NM_H = V_{OH} - V_{IH}$$

$$\text{Now, } V_{IL} = \frac{2V_0 - |V_{TP}| - V_{DD} + kV_{Tn}}{1+k}$$

$$V_{OL} = V_{in} - V_{TP} + \sqrt{(V_{in} - V_{DD} - V_{TP})^2 + k(V_{in} - V_{TP})^2}$$

$$V_{OH} = V_{in} - V_{Tn} + \sqrt{(V_{in} - V_{Tn})^2 + \frac{1}{k}(V_{in} - V_{DD} - V_{TP})^2}$$

$$V_{IH} = \frac{V_{DD} + V_{TP} + k(2V_0 + V_{TP})}{1+k}$$

$$\text{Where } k = \frac{(w/L)_n}{(w/L)_p}$$

\therefore as $W_p \uparrow \rightarrow NM_L \uparrow$ and $NM_H \downarrow$

17. $\nabla \cdot \vec{D} = \rho_v$

This is Gauss law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

This is faraday law of electromagnetic induction

$$\nabla \times \vec{B} = 0$$

This is Gauss law in magnetostatics which states magnetic monopole does not exist.

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

This is modified form of ampere's circuital law.

18. at $F = 10$ Hz we have one pole

At $F = 10^2$ Hz we can see two more poles are added as slope is decreased by 40 dB/decade

At $F = 10^3$ Hz we have a zero

At $F = 10^4$ Hz we have two zero's

At $F = 10^5$ Hz we have two pole's

At $F = 10^6$ we have one pole

\therefore Total poles $N_p = 6$

And total zeros $N_z = 3$

19. $x(t) = \cos(2\pi fct + km(t))$

$\therefore Q(t) = 2\pi fct + km(t)$

And $f_i = \frac{1}{2\pi} \frac{\partial}{\partial t}(Q(t))$

$= \frac{1}{2\pi} \frac{\partial}{\partial t}[2\pi fct + km(t)]$

$f_i = fc + \frac{k}{2\pi} \frac{\partial}{\partial t}m(t)$

$\therefore f_{i_{\max}} = fc + \frac{k}{2\pi} \left[\frac{\partial}{\partial t}m(t) \right]_{\max}$

$\therefore f_{i_{\max}} = 50 \text{ kHz} + 5 \times \frac{1 - (-1)}{(7 - 6) \times 10^{-3}}$

$\therefore f_{i_{\max}} = 50 \text{ kHz} + 10 \text{ kHz}$

$\therefore f_{i_{\max}} = 60 \text{ kHz}$

And $f_{i_{\min}} = fc + \frac{k}{2\pi} \left[\frac{\partial}{\partial t}(m(t)) \right]_{\min}$

$50 \text{ kHz} + 5 \times \frac{-1 - 1}{(9 - 7) \times 10^{-3}}$

$= 50 \text{ kHz} - 5 \text{ kHz}$

$f_{i_{\min}} = 45 \text{ kHz}$

$\therefore \frac{f_{\min}}{f_{\max}} = \frac{45}{60} = 0.75$

20. $D_1 = \bar{Q}_1 \cdot \bar{Q}_2$

$D_2 = Q_1$

Present State		Excitation		Next state	
Q_1	Q_2	D_1	D_2	Q_1^+	Q_2^+
0	0	1	0	1	0
1	0	0	1	0	1
0	1	0	0	0	0

As three states are there

$$\text{Frequency of output} = \text{Frequency of } Q_2 = \frac{12 \text{ kHz}}{3} = 4 \text{ kHz}$$

21. As it is given that it is linear hamming code addition of two codes will produce another code.

(Here we are talking about mod 2 addition)

$$0001 \rightarrow 0000111$$

$$0011 \rightarrow 1100110$$

$$\underline{0010} \rightarrow \underline{1100001}$$

22. Ans. 0367

Sol. Probability density function (Pdf) = $\frac{d}{dx}(\text{CDF})$

$$\therefore \text{Pdf} = \begin{cases} e^{-x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

$$\text{Now } \Pr(z > 2 | z > 1) = \frac{\Pr[(z > 2) \cap (z > 1)]}{\Pr(z > 1)}$$

$$= \frac{\Pr(z > 2)}{\Pr(z > 1)}$$

$$= \frac{\int_2^{\infty} e^{-x} dx}{\int_1^{\infty} e^{-x} dx}$$

$$= \frac{-1(e^{-\infty} - e^{-2})}{-1(e^{-\infty} - e^{-1})}$$

$$= \frac{e^{-2}}{e^{-1}} = \frac{1}{e}$$

$$\therefore \Pr(Z > 2 | Z > 1) = 0.367$$

23. DC value and phase shift does not affect time period of a signal.

So it is equivalent to find time period of

$$x(t) = 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t\right)$$

$$\therefore \omega_1 = \pi \quad T_1 = \frac{2\pi}{\omega_1} = 2 \text{ second}$$

$$\omega_2 = \frac{2\pi}{3} \quad T_2 = \frac{2\pi}{\omega_2} = 3 \text{ second}$$

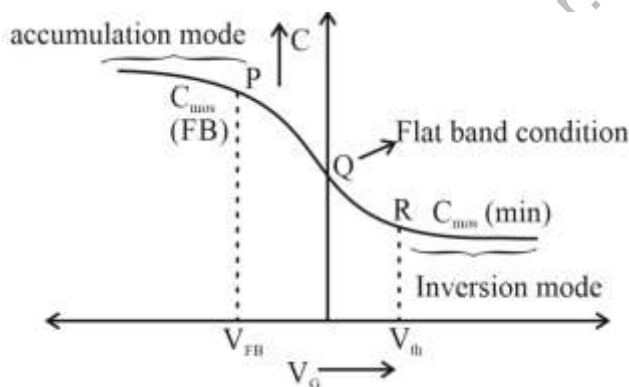
$$\omega_3 = \frac{\pi}{2} \quad T_3 = \frac{2\pi}{\omega_3} = 4 \text{ second}$$

Now overall $T = \text{LCM}(T_1, T_2, T_3)$

$= \text{LCM}(2, 3, 4)$

\therefore overall $T = 12$ seconds

24.



25. $\frac{1}{2\pi i} \oint_{|z|=1} \frac{(z^2 - 1)^2}{z^2} dz$

For poles :

Consider $z^2 = 0 \Rightarrow \boxed{z = 0, 0}$

Now $f(z) = (z^2 + 1)^2$

$$\oint_C \frac{f'(z)}{(z-a)^{n+1}} dz = \frac{2\pi i}{n!} f^{(n)}(a)$$

$$= \frac{1}{2\pi i} \left[\frac{2\pi i}{(2-1)!} f^{(2-1)}(a) \right] = f'(a) = f'(0)$$

Now $f'(z) = 2(z^2 + 1)(2z)$

$$f'(0) = 2(0 + 1)(0) = 0$$

∴ So answer is zero.

26. Let output of MUX is M

$$\text{So } M = \bar{A}\bar{Q} + AQ$$

$$\therefore M = A \odot$$

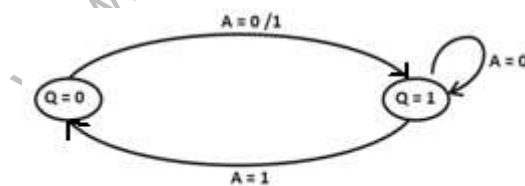
$$\text{And } D = \overline{MQ}$$

$$= \bar{M} + \bar{Q}$$

$$D = A \oplus Q + \bar{Q}$$

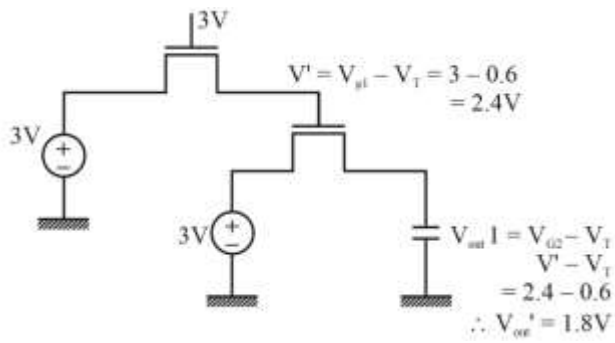
Present State	Input	Next State
Q	A	$Q^+ = D$
0	0	1
0	1	1
1	0	1
1	1	0

State Diagram:-

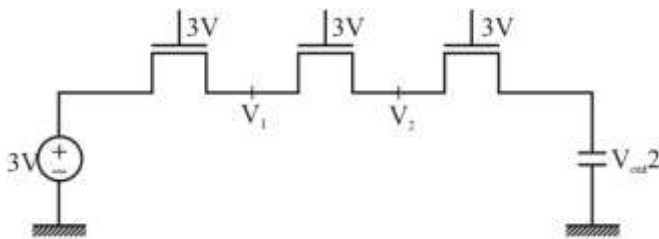


27. Given $V_{TN} = 0.6V$, $V_{SB} = 0$ and $\lambda = 0$

In figure (i)



In figure (ii)



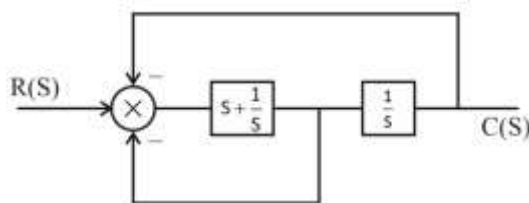
Every MOS transistor has same $V_G = 3V$

$$\therefore V_1 = V_2 = V_{out2} = V_G - V_T$$

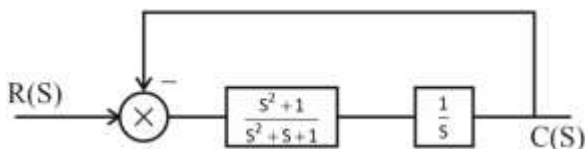
$$= 3 - 0.6$$

$$\therefore V_{out2} = 2.4V$$

28.



$$\frac{\frac{s^2 + 1}{s}}{1 + \frac{s^2 + 1}{s}} = \frac{s^2 + 1}{s^2 + s + 1}$$



$$TF = \frac{\frac{s^2 + 1}{s(s^2 + s + 1)}}{1 + \frac{s^2 + 1}{s(s^2 + s + 1)}}$$

$$\therefore TF = \frac{s^2 + 1}{s^3 + 2s^2 + s + 1}$$

29. $P_0(-1 + N > V_{th})$

$$P_0(N > V_{th} + 1) = \int_{V_{th}+1}^2 \frac{1}{4} dx = \frac{1}{4} [2 - V_{th} - 1] = \frac{1}{4} (1 - V_{th})$$

$$P_1(1 + N < V_{th})$$

$$P_1(N < V_{th} - 1) = \int_{-2}^{V_{th}-1} \frac{1}{4} dx = \frac{1}{4} [V_{th} - 1 + 2] = \frac{1}{4} (V_{th} + 1)$$

$$P_e = P(0)P_0(N > V_{th} + 1) + P(1)P_1(N < V_{th} - 1)$$

$$P_e = 0.2 \times \frac{1}{4} (1 - V_{th}) + 0.8 \times \frac{1}{4} (V_{th} + 1)$$

$$= 0.05 - 0.5V_{th} + 0.2V_{th} + 0.2$$

$$P_e = 0.25 + 0.15V_{th}$$

$$\text{For } V_{th} = 0 \rightarrow P_e = 0.25$$

$$\text{For } V_{th} = 1 \rightarrow P_e = 0.4$$

$$\text{For } V_{th} = -1 \rightarrow P_e = 0.1$$

$$\therefore \text{Minimum probability of error} = 0.1$$

30. Ans. 0.231

Sol. $1 - e^{-\alpha x} = 0.5$

$$e^{-\alpha x} = 0.5$$

$$\text{now } \alpha = 3 \times 10^4 \text{ cm}^{-1}$$

$$\therefore x = \frac{-\ln(0.5)}{3 \times 10^4}$$

31. $I_D = \frac{1}{2} \mu_p C O X \left(\frac{\omega}{L} \right)_p (V_{GSP} - |V_{TP}|)^2$

$$= \frac{1}{2} \times 30 \times 10^{-6} \times 10 \times (2-1)^2$$

$$I_D = 150 \mu A$$

$$\text{Now, } g_m = \sqrt{2I_D \mu_n C_{ox} \left(\frac{W}{L} \right)_N}$$

$$g_m = \sqrt{2 \times 150 \times 10^{-6} \times 60 \times 10^{-6} \times 5}$$

$$\therefore g_m = 300 \times 10^{-6} S$$

$$\text{Now } A_v = -g_m (r_{ds} \parallel r_{ds})$$

$$= -300 \times 10^{-6} \left((6 \times 10^6) \parallel \right)$$

$$= -300 \times 10^{-6} \times 3 \times 10^6$$

$$\therefore A_v = -900$$

32. Given that

$$h(0) = 1, h(1) = a, h(2) = b \text{ and } h(n) = 0 \text{ otherwise}$$

$$\therefore H(e^{jw}) = 1 + ae^{-jw} + be^{-j2w}$$

Now $y(n) = 0$ for all n

$$\text{Now } x(n) = C_1 e^{\left(\frac{-j\pi n}{2} \right)} + C_2 e^{\left(\frac{j\pi n}{2} \right)}$$

If we consider $C_1 e^{\left(\frac{-j\pi n}{2} \right)}$ as input then

$$\text{Output} = C_1 \left[1 + ae^{+j\frac{\pi}{2}} + be^{-j2\left(\frac{\pi}{2} \right)} \right]$$

$$\text{Output} = C_1 \left[1 + ae^{j\frac{\pi}{2}} + be^{j\pi} \right] \quad \dots(i)$$

If we consider $C_2 e^{\left(\frac{j\pi n}{2} \right)}$ as input then

$$\text{Output} = C_2 \left[1 + ae^{-j\frac{\pi}{2}} + be^{-j2\left(\frac{\pi}{2} \right)} \right]$$

$$= C_2 \left[1 + ae^{-j\frac{\pi}{2}} + be^{-j\pi} \right] \quad \dots(ii)$$

Both output (i) and (ii) will be zero if

$$a = 0, \quad b = 1$$

$$33. \quad I_D = \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{\omega}{L} \right) \cdot (V_{gs} - V_T)^2$$

$$= \frac{300 \times 3.45 \times 10^{-7}}{2} \times \left(\frac{10}{1} \right) \times (5 - 0.7)^2$$

$$\therefore I_D = 25.5 \text{ mA}$$

$$34. \text{ Current through FET having } \left(\frac{\omega}{L} \right) = 3 \text{ will be } I_1$$

$$\therefore I_1 = \frac{(\omega/L)_2}{(\omega/L)_1} \times 1 \text{ mA}$$

$$\therefore I_1 = \frac{3}{2} \text{ mA}$$

Now,

$$I_{out} = \frac{(\omega/L)_4}{(\omega/L)_3} \times I_1$$

$$= \frac{40}{10} \times \frac{3}{2} \text{ mA}$$

$$\therefore I_{out} = 6 \text{ mA}$$

$$35. \text{ Quantum Efficiency } \eta = \frac{R_e}{R_p}$$

R_e = Corresponding Electron Rate (electrons/sec)

R_p = Incident Photon Rate (Photons/sec)

$$R_e = \frac{I_p}{q}, \quad R_p = \frac{P_{in}}{h\nu}, \quad \eta = \frac{I_p}{P_{in}}$$

$$\eta = \frac{I_p/q}{P_{in}/h\nu}$$

Now

$$\eta = \frac{I_p/q}{P_{in}/h\nu} = \frac{I_p h\nu}{q P_{in}} = \frac{h\nu R}{q}$$

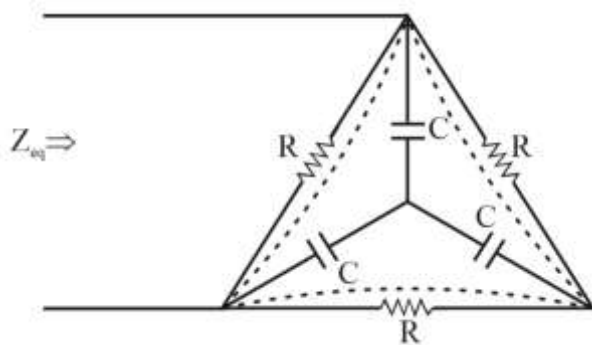
So

$$\Rightarrow R = \frac{q\eta}{h\nu} = \frac{q\eta\lambda}{hc} = \eta \times \left(\frac{q}{hc} \right)$$

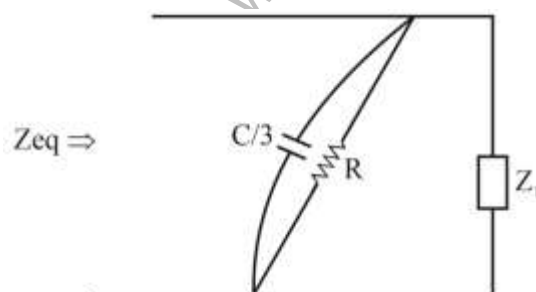
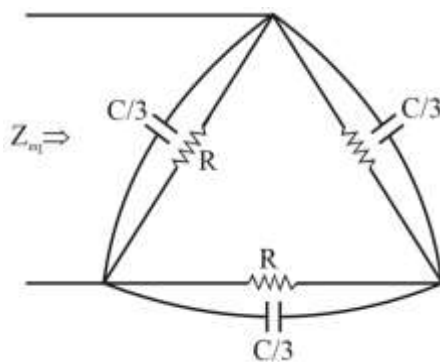
$$q = 1.6 \times 10^{-19} \text{C}, h = 6.63 \times 10^{-34} \text{Js}, C = 3 \times 10^8 \text{ m/s}$$

$$R = \frac{\eta\lambda}{1.24}$$

36.



Performing star to delta conversion



$$\text{Where } Z_1 = 2 \left[\frac{R}{1 + \frac{j\omega CR}{3}} \right]$$

$$\therefore Z_{eq} = Z_1 \parallel \left(\frac{R}{1 + \frac{j\omega CR}{3}} \right)$$

$$\therefore Z_{eq} = \frac{2}{3} \left(\frac{R}{1 + \frac{j\omega CR}{3}} \right)$$

Now $R = 1\text{ kW}$, $C = 1\mu\text{F}$ and $\omega = 1000 \text{ rad/sec}$

$$\therefore Z_{eq} = 0.66 - 0.2178j$$

$$\therefore I = \frac{V}{Z_{eq}}$$

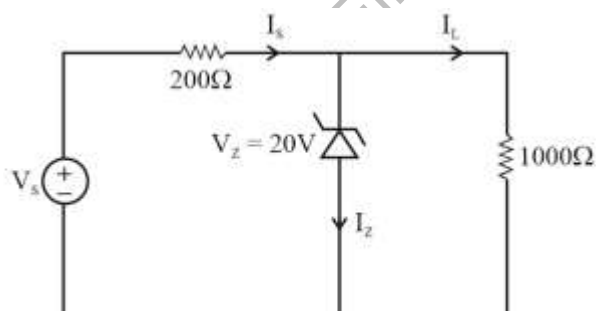
$$= \frac{2 \sin(1000t)}{0.66 - 0.2178j}$$

$$= \frac{2}{\sqrt{0.66^2 + 0.2178^2}} \cdot \sin \left(1000t - \tan^{-1} \left(\frac{1}{3} \right) \right)$$

$$= 3.16 \sin(1000t + 18.43^\circ)$$

$$\therefore I \approx 3 \sin(1000t) + \cos(1000t)$$

37.



$$I_{z\text{max}} = 60 \text{ mA}$$

$$I_L = \frac{20}{1000} = 20\text{mA}$$

As I_{Zmin} not given,

$$I_{Zmin} = 0 \text{ mA}$$

$$\text{Now } I_S = I_Z + I_L$$

$$\therefore I_{Smin} = I_{Zmin} + I_L$$

$$= 0 + 20 \text{ mA}$$

$$\therefore I_{Smin} = 20 \text{ mA}$$

$$\text{Now } I_S = \frac{V_S - V_Z}{200}$$

$$\therefore 20\text{mA} = \frac{V_S - 20}{200}$$

$$\therefore V_S = 24\text{V}$$

$$\text{Now } I_{Smax} = I_{Zmax} + I_L$$

$$= 60 + 20$$

$$I_{Smax} = 80 \text{ mA}$$

$$\therefore I_S = \frac{V_S - V_Z}{200}$$

$$\therefore 80\text{mA} = \frac{V_S - 20}{200}$$

$$\therefore V_S = 36 \text{ V}$$

38.

Sol. $H = \frac{I}{2\pi\rho} a\rho$

For wire ω_1

$$H_1 = \frac{I}{2\pi r}$$

For wire ω_2

$$H_2 = \frac{2I}{2\pi 3r}$$

Magnetic field will be circular and can be find out by right hand rule

Both fields will add at middle region

\therefore at dotted line

$$H = H_1 + H_2$$

$$\therefore H = \frac{5I}{6\pi r}$$

$$\text{Now } B = \mu_0 H$$

$$B = \frac{\mu_0 5I}{6\pi r}$$

39.

Sol. $V_g = \frac{d\omega}{d\beta}$

$$\text{Now, } \frac{d\beta}{d\omega} = \frac{dk(\omega)}{d\omega} = \frac{d}{d\omega} \cdot \frac{1}{c} \sqrt{\omega^2 - \omega_0^2} = \frac{1}{2c\sqrt{\omega^2 - \omega_0^2}} \times 2\omega$$

$$\boxed{\frac{d\beta}{d\omega} = \frac{\omega}{c\sqrt{\omega^2 - \omega_0^2}}}$$

$$V_g = \frac{1}{\frac{\omega}{c\sqrt{\omega^2 - \omega_0^2}}} = 2 \times 10^8 \Rightarrow \frac{c\sqrt{\omega^2 - \omega_0^2}}{\omega} = 2 \times 10^8$$

$$\Rightarrow \boxed{\sqrt{\omega^2 - \omega_0^2} = \frac{2\omega}{3}}$$

$$\text{Now, } V_p = \frac{\omega}{\beta} = \frac{\omega}{k} = \frac{\omega}{\frac{1}{c}\sqrt{\omega^2 - \omega_0^2}} = \frac{\omega c}{\frac{2\omega}{3}} = \frac{3c}{2}$$

$$V_p = \frac{3}{2} \times 3 \times 10^8 = 4.5 \times 10^8 \text{ m/s}$$

$$\boxed{V_p = 4.5 \times 10^8 \text{ m/s}}$$

40. $f(-1) = 0$

So only option (B) and (C) are possible

Let's try option (B)

$$f(x) = 2|x + 1|$$

$$\therefore f(x) = \begin{cases} 2(x+1) & \text{for } x+1 > 0 \\ -2(x+1) & \text{for } x+1 < 0 \end{cases}$$

$$\therefore f(x) = \begin{cases} 2(x+1) & \text{for } x > -1 \\ -2(x+1) & \text{for } x < -1 \end{cases}$$

$$\therefore f'(x) = \begin{cases} 2 & \text{for } x > -1 \\ -2 & \text{for } x < -1 \end{cases}$$

$$\therefore |f'(x)| \leq 2$$

\therefore option (B) is correct.

41. $G(s) = \frac{C(s)}{R(s)}$

$$\therefore C(s) = G(s) \cdot R(s)$$

$$= \frac{1}{s(s^2 + 2s + 1)}$$

$$\therefore C(s) = \frac{1}{s(s+1)^2}$$

$$\therefore C(s) = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2}$$

$$\therefore A(s+1)^2 + Bs(s+1) + Cs = 1$$

$$\therefore As^2 + 2As + A + Bs^2 + Bs + Cs = 1$$

$$\therefore A + B = 0$$

$$\therefore 2A + B + C = 0$$

$$\therefore A = 1$$

$$\text{So } B = -1$$

$$\text{And } C = -1$$

$$\therefore C(s) = \frac{1}{s} + \frac{-1}{s+1} + \frac{-1}{(s+1)^2}$$

$$\therefore C(t) = (1 - e^{-t} - te^{-t}) u(t)$$

At $t \rightarrow \infty$ steady state will occur

$$\therefore C(\infty) = 1$$

Now we are asked to find time at which 94% of the steady state value reached.

$$\therefore C(t) = 1 - e^{-t} - te^{-t} = 0.94$$

$$\therefore e^{-t} + te^{-t} = 0.06$$

$$\therefore e^{-t}(1+t) = 0.06$$

Now from the given options try all option you will get $t = 4.50$ sec.

42.
$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

We are obtaining $X(1)$ correctly

$$\therefore k = 1$$

$$\therefore x(1) = x(0) + x(1)W_6^1 + x(2)W_6^2 + x(3)W_6^3 + x(4)W_6^4 + x(5)W_6^5$$

We know that

$$W_N^{k+\frac{N}{2}} = -W_N^k$$

$$\therefore W_6^3 = -W_6^0 = -1$$

$$W_6^4 = -W_6^1$$

$$W_6^5 = -W_6^2$$

\therefore comparing with given graph

$$a_1 = 1, \quad a_2 = W_6, \quad a_3 = W_6^2$$

43.
$$H(s) = \frac{1}{s^2 + 3s^2 + 2s + 1}$$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [u]$$

$$\& [y] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} [u]$$

$$\therefore A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \text{ and } C = [1 \ 0 \ 0]$$

44. Same current will flow through both NMOS & PMOS

$$\therefore I_{D1} = I_{D2}$$

$$\therefore \frac{\mu_n C_{OX}}{2} \cdot \left(\frac{\omega}{L}\right)_N (V_{GSN} - V_{TN})^2 = \frac{\mu_p C_{OX}}{2} \cdot \left(\frac{\omega}{L}\right)_p (V_{GSP} - |V_{TP}|)^2$$

$$\therefore 100 \times \left(\frac{\omega}{L}\right)_N \cdot (1.5 - 0.7)^2 = 400 \times \left(\frac{\omega}{L}\right)_p (1.5 - 0.9)^2$$

$$\therefore \frac{(\omega/L)_N}{(\omega/L)_p} = \frac{9}{16} \times \frac{4}{10}$$

$$= 0.225$$

$$\left(\because I_{GSP} = \frac{V_{dd}}{2} = 1.5V \right)$$

45.

$$f_c = \frac{V}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\text{For } T\epsilon_{10}, \quad m = 1, \quad n = 0$$

$$f_{c1} = \frac{V}{2} \sqrt{\left(\frac{1}{a}\right)^2} = 0 = \frac{V}{2a}$$

$$\text{For } T\epsilon_{11}, \quad m = 1, \quad n = 1$$

$$f_{c2} = \frac{V}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$$

Given $\frac{f_{c1}}{f_{c2}} = \frac{1}{2}$

$$\frac{V/2a}{\frac{V}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{1}{2}$$

$$\frac{\frac{1}{a}}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{1}{2} \Rightarrow \frac{\frac{1}{a}}{\frac{\sqrt{a^2 + b^2}}{ab}} = \frac{1}{2}$$

$$\frac{b}{\sqrt{a^2 + b^2}} = \frac{1}{2}$$

$$\Rightarrow 4b^2 = a^2 + b^2$$

$$\Rightarrow 3b^2 = a^2$$

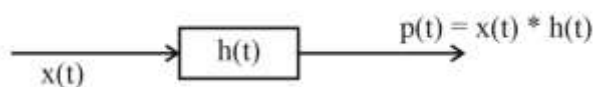
$$\Rightarrow \frac{b^2}{a^2} = \frac{1}{3}$$

$$\Rightarrow \frac{b}{a} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{a}{b} = \sqrt{3}$$

$$\frac{\text{width}}{\text{height}} = \sqrt{3} = 1.732$$

46.



and $y(t) = z(t) + p(t)$

$$\therefore R_{yy}(\tau) = R_{zz}(\tau) + R_{pp}(\tau) + R_{pz}(\tau) + R_{zp}(\tau)$$

now $x(t)$ & $z(t)$ are uncorrelated.

$$\therefore R_{pz}(\tau) = R_{zp}(\tau) = 0$$

$$\therefore R_{yy}(\tau) = R_{zz}(\tau) + R_{pp}(\tau)$$

So the power spectral relation can be given by Fourier transform of the above relation.

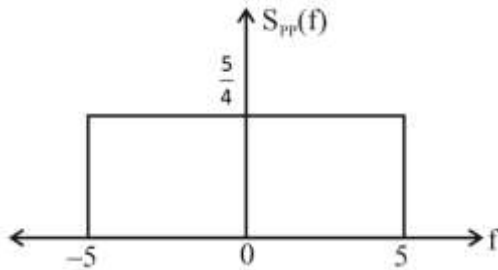
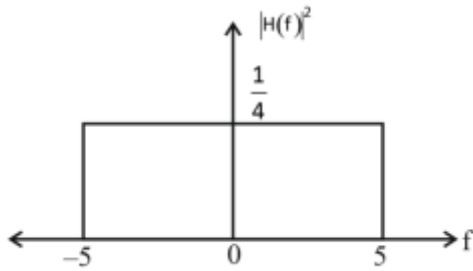
$$\therefore S_{yy}(f) = S_{zz}(f) + S_{pp}(f)$$

now power of $y(t)=$

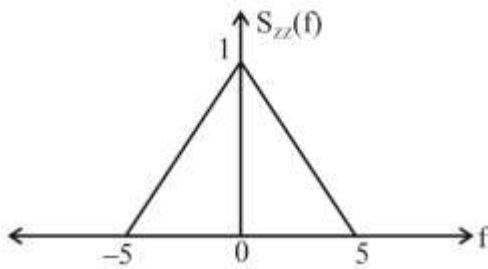
$$\int_{-\infty}^{\infty} S_{yy}(f) df$$

$$\therefore P = \int_{-\infty}^{\infty} S_{zz}(f) df + \int_{-\infty}^{\infty} S_{pp}(f) df$$

$$\text{now } S_{pp}(f) = |H(f)|^2 \times S_{xx}(f)$$



&



$$\therefore P = \frac{5}{4} \times 10 + \frac{1}{2} \times 10 \times 1$$

$$\therefore P = 17.5 \text{ watt}$$

47. For the minimization of the energy in the error signal there are different approaches like, Prony's method, Pade approximation. As $g(n)$ has three samples.

Consider them as $g(-1)$, $g(0)$, $g(1)$ we can minimise $E(h,g)$ by making $h(n) = g(n)$ using rectangular window and Parseval's theorem of OTFT.

Based on which $10g(-1) + g(1) = 10(-3) + 3$

$$= -27$$

48. $I_r = 0.75 I_s$

$$\therefore \text{Forward current} = I_D = -0.75 I_s$$

$$\therefore I_s(e^{v_0/nvT} - 1) = -0.75 I_s$$

Now Take $n = 1$

$$\therefore e^{v_0/vT} = 0.25$$

$$\therefore V_D = V T \ln(0.25)$$

$$\therefore V_R = -V T \ln(0.25)$$

$$= -\frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times -1.386$$

$$\therefore V_R = 35.87 \text{ mv}$$

49. Given differential equation is of Cauchy – Euler differential equation type.

So let $x = e^z \quad \therefore z = \ln x$

The differential equation can be written as,

$$D(D-1) - 3D + 3 = 0$$

$$\therefore D^2 - 4D + 3 = 0$$

$$\therefore D = 1, 3$$

$$\therefore y = C_1 e^z + C_2 e^{3z}$$

$$\therefore y = C_1 x + C_2 x^3$$

Now $y(1) = 1$

$$\therefore C_1 + C_2 = 1 \quad \dots(i)$$

And $y(2) = 14$

$$\therefore 2C_1 + 8C_2 = 14 \quad \dots(ii)$$

From (i) and (ii)

$$C_1 = -1, C_2 = 2$$

$$\therefore y = -x + 2x^3$$

$$\therefore y(1.5) = -1.5 + 2(1.5)^3$$

$$\therefore y(1.5) = 5.25$$

50. We know that,

$$I_C(t) = C \frac{dV_C(t)}{dt}$$

And capacitor will be charged by the following equation

$$V_C(t) = V_S(1 - e^{-t/\tau})$$

$$I_C(t) = C \cdot \frac{d}{dt} \left[V_S(1 - e^{-t/\tau}) \right]$$

$$\therefore I_C(t) = \frac{V_S}{R(t)} e^{-t/R(t) \cdot C}$$

$$\text{Given } R(t) = R_0 \left[1 - \frac{t}{T} \right]$$

Now $R_0 = 1$ and $C = 1$

$$\therefore T = 3R_0C = 3$$

$$\therefore R(t) = \left[1 - \frac{t}{3} \right]$$

$$\& I_C(t) = \frac{1}{\left(1 - \frac{t}{3} \right)} \times e^{\left(\frac{-t}{1 - \frac{t}{3}} \right)}$$

$$\text{At } t = \frac{T}{2} = \frac{3}{2} \text{ sec}$$

$$I_C(t) = 2e^{-3}$$

$$= 0.099$$

$$I_C(t) \approx 0.1 \text{ mA}$$

51. $V_S = 10 \text{ V}$

Voltage across capacitor will be

$$V_C(t) = 10(1 - e^{-t/RC})$$

$$R_C = 500 \times 10 \times 10^{-6} = 5 \times 10^{-3} \text{ sec}$$

$$\text{At } t = 2 \text{ ms} = 2 \times 10^{-3} \text{ sec}$$

And for roots on imaginary axis $s^1 \text{ row} = 0$

$$\therefore \frac{6-k}{3} = 0$$

$$\therefore k = 6$$

54.

$m(t)$ has frequency range 5 kHz to 15 kHz

Now it is amplitude modulated

$$f(t) = A (1 + m(t)) \cos 2\pi f_c t \text{ where } f_c = 600 \text{ kHz}$$

$$\therefore \text{AM signal will have highest frequency} = f_c + f_m (\text{max})$$

$$= 600 + 15 = 615 \text{ kHz}$$

$$\text{And AM signal will have lowest frequency} = f_c - f_m (\text{max})$$

$$= 600 - 15 = 585 \text{ kHz}$$

It is a band pass signal so we use bandpass sampling

$$f_s = 1.2 \times \frac{2f_H}{K}$$

$$K = \frac{f_H}{f_H - f_L}$$

$$= \frac{615}{615 - 585}$$

$$K = 20.5$$

We select $K = 20$

$$\therefore f_s = 1.2 \times \frac{2 \times 615}{20}$$

$$\therefore f_s = 73.8 \text{ kHz}$$

$$\text{Now } L = 256$$

$$\text{And } 2^n = L = 256$$

$$\therefore n = 8$$

$$\text{Bitrate} = R_b = n f_s$$

$$\therefore R_b = 8 \times 73.8 \times 10^3$$

$$\therefore R_b = 0.59 \text{ Mbps}$$

55.

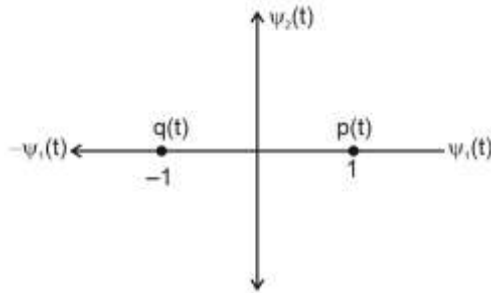
0 is represented by $p(t)$

And 1 is represented by $q(t)$

And $\psi_1(t)$ and $\psi_2(t)$ are orthogonal signal set

(i) $p(t) = \psi_1(t)$ and $q(t) = -\psi_1(t)$

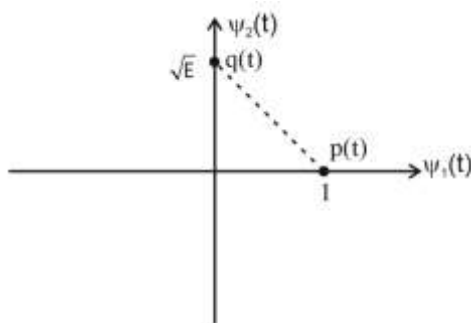
So signal space diagram will be,



$$\therefore d_{min_1} = 2$$

(ii) $p(t) = \psi_1(t)$ and $q(t) = \sqrt{E}\psi_2(t)$

So signal space diagram will be



$$\therefore d_{min_2} = \sqrt{1+E}$$

Now bit error probability is same in both cases

$$\therefore d_{min_1} = d_{min_2}$$

$$\sqrt{1+E} = 2$$

$$\therefore E = 3$$