DU MPhil Phd in Mathematics

Topic:- DU_J19_MPHIL_MATHS

1) Which of the following journals is published by Indian Mathematical Society

[Question ID = 13918]

1. Indian Journal of Pure and Applied Mathematics. [Option ID = 25669]
2. Indian Journal of Mathematics. [Option ID = 25671]
3. Ramanujan Journal of Mathematics. [Option ID = 25670]
4. The Mathematics Students . [Option ID = 25672]

Correct Answer :-

- Indian Journal of Pure and Applied Mathematics. [Option ID = 25669]

2) Name a Fellow of Royal Society who expired in 2019 [Question ID = 13917]

1. M. S. Ragunathan. [Option ID = 25665]
2. Manjul Bhargava. [Option ID = 25666]
3. Michael Atiyah. [Option ID = 25667]
4. S. R. Srinivasa Varadhan. [Option ID = 25668]

Correct Answer :-

- M. S. Ragunathan. [Option ID = 25665]

3) Which of the following statements is true? [Question ID = 13973]

1. Every topological space having Bolzano-Weiestrass property is a compact space. [Option ID = 25890]
2. If $\{x_n\}$ is a convergent sequence in a topological space X with a limit x then $Y = \{x\} \cup \{x_n : n = 1, 2, \cdots\}$ is a compact subset of X. [Option ID = 25891]
3. The projection map $p : X \times Y \to Y$ defined by $p(x, y) = y$ is a closed map for all topological spaces X, Y. [Option ID = 25889]
4. Every topological space is a first countable space. [Option ID = 25892]

Correct Answer :-

- The projection map $p : X \times Y \to Y$ defined by $p(x, y) = y$ is a closed map for all topological spaces X, Y. [Option ID = 25889]

4) Which of the following statements is true for topological spaces? [Question ID = 13927]

1. Every second countable space is separable. [Option ID = 25706]

2. Every separable space is second countable. [Option ID = 25705]

3. Every first countable space is second countable. [Option ID = 25708]
4. Every first countable space is separable. [Option ID = 25707]

Correct Answer :-

- Every separable space is second countable. [Option ID = 25705]

5) Which of the following statements is not true? [Question ID = 13997]

1. If H and K are normal subgroups of G, then the subgroup generated by $H \cup K$ is also a normal subgroup of G.
   [Option ID = 25987]
2. Let G be a finite group and H a subgroup of order n. If H is the only subgroup of order n, then H is normal in G.
   [Option ID = 25986]
3. The set of all permutations σ of $S_n\,(n \geq 3)$ such that $\sigma(n) = n$ is a normal subgroup of S_n.
   [Option ID = 25985]
4. For groups G and H and $f : G \to H$ a group homomorphism. If H is abelian and V is a subgroup of G containing $\ker f$ then N is a normal subgroup of G.
   [Option ID = 25988]

Correct Answer :-

- The set of all permutations σ of $S_n\,(n \geq 3)$ such that $\sigma(n) = n$ is a normal subgroup of S_n.
  [Option ID = 25985]

6) Which one of the following fellowship is based on merit in M.A/M.Sc. of the University [Question ID = 13920]

1. NBHM-JRF. [Option ID = 25679]
2. INSPIRE-JRF [Option ID = 25677]
3. UGC-JRF. [Option ID = 25680]
4. CSIR-JRF [Option ID = 25678]

Correct Answer :-

- INSPIRE-JRF [Option ID = 25677]

7) The Abel prize 2019 was awarded to [Question ID = 13919]

1. Lennert Carleson. [Option ID = 25673]
2. Mikhail Gromov. [Option ID = 25676]
3. Karen Keskulla Uhlenbeck. [Option ID = 25674]
4. Peter Lax. [Option ID = 25675]

Correct Answer :-

- Lennert Carleson. [Option ID = 25673]

Let X be a normed space over $\mathbb{C}$ and f a non-zero linear functional on X. Then

[Question ID = 13981]

1. f is surjective and a closed map. [Option ID = 25922]
2. f is surjective and open. [Option ID = 25921]
3. f is continuous and bijective. [Option ID = 25924]
4. f is open and continuous. [Option ID = 25923]

Correct Answer :-

- f is surjective and open. [Option ID = 25921]

9)

Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$.

Then which of the following statements is not true?

[Question ID = 13968]

1. f is bounded above on (a, ∞). [Option ID = 25869]
2. f' is not continuous at 0. [Option ID = 25871]
3. f is infinitly differentiable at every non zero $x \in \mathbb{R}$. [Option ID = 25870]
4. f is neither convex nor concave on $(0, \delta)$. [Option ID = 25872]

Correct Answer :-

- f is bounded above on (a, ∞). [Option ID = 25869]

10)

The principal part of the Laurent series of $f(z) = \dfrac{1}{z(z-1)(z-3)}$ in the annulus $\{z : 0 < |z| < 1\}$ is

[Question ID = 13988]

1. $-\dfrac{1}{3z}$ [Option ID = 25951]
2. $\dfrac{1}{z}$. [Option ID = 25949]
3. $\dfrac{1}{3z}$. [Option ID = 25952]
4. $\dfrac{1}{3z^2}$. [Option ID = 25950]

Correct Answer :-

- $\frac{1}{z}$. [Option ID = 25949]

11) The general solution of the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \cot\frac{y}{x}$$

where c is a constant, is

[Question ID = 14009]

1. cosec(y/x) = c/x. [Option ID = 26036]
2. cosec(y/x) = cx. [Option ID = 26035]
3. sec(y/x) = cx. [Option ID = 26033]
4. sec(y/x) = c/x. [Option ID = 26034]

Correct Answer :-

- sec(y/x) = cx. [Option ID = 26033]

12)

Velocity potential for the uniform stream flow with velocity $\overline{q} = -Ui$, where U is constant and i is the unit vector in x-direction, past a stationary sphere of radius a and centre at origin, for $r \geq a$ is

[Question ID = 14008]

1. $U\cos\theta\,(r + \frac{1}{2}\frac{a^2}{r^3})$. [Option ID = 26029]
2. $U\cos\theta\,(r^2 + \frac{a^2}{r^3})$ [Option ID = 26032]
3. $U\cos\theta\,(r^2 + \frac{1}{2}\frac{a^2}{r^3})$ [Option ID = 26031]
4. $U\cos\theta\,(r + \frac{a^2}{r^3})$ [Option ID = 26030]

Correct Answer :-

- $U\cos\theta\,(r + \frac{1}{2}\frac{a^2}{r^3})$. [Option ID = 26029]

13)

Let $X = P[a, b]$ be the linear space of all polynomials on $[a, b]$. Then which of the following statements is not true?

[Question ID = 13979]

1. X is dense in $C[a, b]$ with $||\,.\,||_p$-norm, $1 \leq p \leq \infty$. [Option ID = 25916]
2. X is a Banach space with $||\,.\,||_p$- norm, $1 \leq p \leq \infty$. [Option ID = 25913]
3. X has a denumerable basis. [Option ID = 25915]
4. X is incomplete with $||\,.\,||_\infty$-norm. [Option ID = 25914]

Correct Answer :-

- X is a Banach space with $\|\cdot\|_p$- norm, $1 \leq p \leq \infty$. [Option ID = 25913]

14)

Let $W = \{(x, x, x) : x \in \mathbb{R}\}$ be a subspace of the inner product space $\mathbb{R}^3$ over $\mathbb{R}$. The orthogonal complement of W in $\mathbb{R}^3$ is the plane

[Question ID = 13995]

1. 2x + y + z = 0. [Option ID = 25979]
2. x + 2y + z = 0. [Option ID = 25978]
3. x + y + z = 0. [Option ID = 25980]
4. x + y + 2z = 0. [Option ID = 25977]

Correct Answer :-

- x + y + 2z = 0. [Option ID = 25977]

15)

The integral surface of the partial differential equation $x^2p + y^2q + z^2 = 0$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ which passes through the hyperbola $xy = x + y$, $z = 1$ is

[Question ID = 14007]

1. $\frac{1}{x} + \frac{2}{y} + \frac{1}{z} = 3.$ [Option ID = 26027]
2. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 3.$ [Option ID = 26028]
3. $\frac{2}{x} + \frac{1}{y} + \frac{1}{z} = 3.$ [Option ID = 26026]
4. $\frac{1}{x} + \frac{1}{y} + \frac{2}{z} = 3.$ [Option ID = 26025]

Correct Answer :-

- $\frac{1}{x} + \frac{1}{y} + \frac{2}{z} = 3.$ [Option ID = 26025]

16)

The value of $\oint_C x^2dx + (xy + y^2)dy$, where C is the boundary of the region R bounded by $y = x$ and $y = x^2$ and is oriented in positive direction is

[Question ID = 13969]

1. 1/15 [Option ID = 25876]
2. 2 [Option ID = 25875]
3. 1/10 [Option ID = 25874]
4. 1/5 [Option ID = 25873]

Correct Answer :-

- 1/5 [Option ID = 25873]

17)

Let $W = \{(x, y, 0) : x, y \in \mathbb{R}\}$ be a subspace of $\mathbb{R}^3$. The cosets of W in $\mathbb{R}^3$ are

[Question ID = 13994]

1. lines parallel to z-axis. [Option ID = 25975]
2. lines perpendicular to z-axis. [Option ID = 25976]
3. planes perpendicular to xz- plane. [Option ID = 25973]
4. planes parallel to yz- plane. [Option ID = 25974]

Correct Answer :-

- planes perpendicular to xz- plane. [Option ID = 25973]

18)

Let R be a ring with unity. An element a of R is called nilpotent if $a^n = 0$ for some positive integer n. An element a of R is called unipotent if and only if $1 - a$ is nilpotent. Consider the following statements:

(I) In a commutative ring with unity, product of two unipotent elements is invertible.
(II) In a ring with unity, every unipotent element is invertible.
Then

[Question ID = 14001]

1. Neither (I) nor (II) is correct. [Option ID = 26004]
2. Both (I) and (II) are correct. [Option ID = 26003]
3. Only (I) is correct. [Option ID = 26001]
4. Only (II) is correct. [Option ID = 26002]

Correct Answer :-

- Only (I) is correct. [Option ID = 26001]

19) Which of the following statements is not true?

[Question ID = 13970]

1. $g_n(x) = \dfrac{1}{n(1+x^2)} \to 0,\ n \to \infty$ uniformly on $\mathbb{R}$. [Option ID = 25877]
2. $h_n(x) = \dfrac{\sin nx}{n}$ converges uniformly on $\mathbb{R}$. [Option ID = 25879]
3. $f_n(x) = \dfrac{x^2 + nx}{x}$ converges uniformly on $\mathbb{R}$. [Option ID = 25878]
4. $u_n(x) = \dfrac{x^n}{n}$ converges uniformly on $[0,\ 1]$. [Option ID = 25880]

Correct Answer :-

$g_n(x) = \frac{1}{n(1+x^2)} \to 0,\ n \to \infty$ uniformly on $\mathbb{R}$.

- [Option ID = 25877]

20)

The value of the integral $\int_C \frac{dz}{z^2+4}$ where C is the anticlockwise circle $|z-i|=2$ is

[Question ID = 13984]

1. 2π. [Option ID = 25935]
2. 0 [Option ID = 25933]
3. π/2. [Option ID = 25934]
4. π. [Option ID = 25936]

Correct Answer :-

- 0 [Option ID = 25933]

21)

Which of the following statements is true for the product $\prod_{\alpha\in\Lambda} X_\alpha$ with product topology of a family $\{X_\alpha\}_{\alpha\in\Lambda}$ of topological spaces?

[Question ID = 13974]

1. If each X_α is metrizable then $\prod_{\alpha\in\Lambda} X_\alpha$ is metrizable. [Option ID = 25895]
2. If each X_α is normal then $\prod_{\alpha\in\Lambda} X_\alpha$ is normal. [Option ID = 25893]
3. If each X_α is completely regular then $\prod_{\alpha\in\Lambda} X_\alpha$ is completely regular. [Option ID = 25896]
4. If each X_α is locally connected then $\prod_{\alpha\in\Lambda} X_\alpha$ is locally connected. [Option ID = 25894]

Correct Answer :-

- If each X_α is normal then $\prod_{\alpha\in\Lambda} X_\alpha$ is normal. [Option ID = 25893]

22) Consider $\mathbb{R}$ with usual metric and a continuous map $f:\mathbb{R}\to\mathbb{R}$ then

[Question ID = 13975]

1. $f(A)$ is bounded for every bounded subset A of $\mathbb{R}$. [Option ID = 25899]
2. f is bounded. [Option ID = 25897]
3. $f^{-1}(A)$ is compact for all compact subset A of $\mathbb{R}$. [Option ID = 25900]
4. Image of f is an open subset of $\mathbb{R}$. [Option ID = 25898]

Correct Answer :-

- f is bounded. [Option ID = 25897]

23) Define a sequence of functions $\{f_n\}$ on $\mathbb{R}$ as

$$f_n(x) = \begin{cases} 1, & \text{if } x \in [-n-2, -n) \\ 0, & \text{otherwise.} \end{cases}$$

Let $\alpha = \int_{-\infty}^{\infty} \lim_{n\to\infty} f_n(x)dx$ and $\beta = \lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(x)dx$. Then

[Question ID = 13986]

1. $0 < \alpha < 1, \beta = 1$ [Option ID = 25942]
2. $\alpha = 0, \beta = \infty.$ [Option ID = 25943]
3. $\alpha = \beta = 0.$ [Option ID = 25941]
4. $\alpha = 0, \beta = 2.$ [Option ID = 25944]

Correct Answer :-

- $\alpha = \beta = 0.$ [Option ID = 25941]

24)

Suppose f is an entire function with $f(0) = 0$ and u be the real part of f such that $|u(x, y)| \leq 1$ for all $(x, y) \in \mathbb{R}^2$. Then the range of u is

[Question ID = 13985]

1. $[-1, 1]$. [Option ID = 25938]
2. $[0, 1]$. [Option ID = 25937]
3. $\{0\}$. [Option ID = 25939]
4. $[-1, 0]$. [Option ID = 25940]

Correct Answer :-

- $[0, 1]$. [Option ID = 25937]

25)

For the minimal splitting field F of a polynomial $f(x)$ of degree n over a field K. Consider the following statements:

(I) F over K is a normal extension.

(II) $n | [F : K]$.

(III) F over K is a separable extension.

Then

[Question ID = 14002]

1. All (I), (II) and (III) are true. [Option ID = 26007]
2. None of (I), (II)and (III) is true. [Option ID = 26008]
3. Only (I) is true. [Option ID = 26005]
4. Only (I) and (II) are true. [Option ID = 26006]

Correct Answer :-

- Only (I) is true. [Option ID = 26005]

26)

Let $V = \{x + \alpha y : \alpha, x, y \in \mathbb{Q}\}$. Then V a vector space over $\mathbb{Q}$ of dimension

[Question ID = 13991]

1. 2 [Option ID = 25963]
2. 1 [Option ID = 25964]
3. 3 [Option ID = 25962]
4. infinity. [Option ID = 25961]

Correct Answer :-

- infinity. [Option ID = 25961]

27)

Let $X = \mathbb{C}^2$ with $||\,.\,||_1$ norm and $X_0 = \{(x_1, x_2) \in X : x_2 = 0\}$. Define $g : X_0 \to \mathbb{C}$ by $g(x) = x_1$, $x = (x_1, 0)$. Consider the following statements:

(I) Every $f \in X'$ (dual space of X) is of the form $f(x_1, x_2) = ax_1 + bx_2$ for some $a, b \in \mathbb{C}$.

(II) Hahn-Banach extensions of g are precisely of the form $f(x) = x_1 + bx_2$, $x = (x_1, x_2) \in X$, $|b| \leq 1$, $b \in \mathbb{C}$.

Then

[Question ID = 13982]

1. (I) is true but (II) is false. [Option ID = 25925]
2. (I) is false but (II) is true. [Option ID = 25926]
3. Neither (I) nor (II) is true. [Option ID = 25927]
4. Both (I) and (II) are true. [Option ID = 25928]

Correct Answer :-

- (I) is true but (II) is false. [Option ID = 25925]

28)

Which of the following statements is not true for a subset A of a metric space X, whose closure is $\overline{A}$?

[Question ID = 13978]

1. If X is totally bounded then A is totally bounded. [Option ID = 25911]
2. A is connected if and only if $\overline{A}$ is connected. [Option ID = 25912]
3. A is bounded if and only if $\overline{A}$ is bounded. [Option ID = 25909]
4. A is totally bounded if and only if $\overline{A}$ is totally bounded. [Option ID = 25910]

Correct Answer :-

- A is bounded if and only if $\overline{A}$ is bounded. [Option ID = 25909]

29) How many pairs of elements are there that generate

$$D_8 = \langle a, b \,|\, a^4 = b^2 = 1,\ ab = ba^{-1} \rangle$$

[Question ID = 13998]

1. 2 [Option ID = 25989]
2. 5 [Option ID = 25991]
3. 8 [Option ID = 25992]
4. 4 [Option ID = 25990]

Correct Answer :-

- 2 [Option ID = 25989]

30)

For each $n \in \mathbb{N}$, define $x_n \in C[0,1]$ by

$$x_n(t) = \begin{cases} n^2 t, & 0 \le t \le 1/n \\ 1/t, & 1/n < t \le 1 \end{cases}$$

where $C[0,1]$ is endowed with sup-norm. Then which of the following is not true:

[Question ID = 13983]

1. The sequence $\{x_n\}_{n\in\mathbb{N}}$ is uniformly bounded on $[0,1]$. [Option ID = 25931]
2. Each x_n is uniformly continuous on $[0,1]$. [Option ID = 25932]
3. The set $\{x_n(t) : n \in \mathbb{N}\}$ is bounded for each $t \in [0,1]$. [Option ID = 25929]
4. $||x_n||_\infty \le n$ for all n. [Option ID = 25930]

Correct Answer :-

- The set $\{x_n(t) : n \in \mathbb{N}\}$ is bounded for each $t \in [0,1]$. [Option ID = 25929]

31)

The eigenvalues of the boundary value problem $y'' + y' + (1+\lambda)y = 0$, $y(0) = 0$, $y(1) = 0$ are

[Question ID = 14005]

1. $-\frac{3}{4} + n^2$, $n \in \mathbb{N}$. [Option ID = 26018]
2. $\frac{3}{4} + n^2\pi^2$, $n \in \mathbb{N}$. [Option ID = 26019]
3. $-\frac{3}{4} + n^2\pi^2$, $n \in \mathbb{N}$. [Option ID = 26020]
4. $\frac{3}{4} + n^2$, $n \in \mathbb{N}$. [Option ID = 26017]

Correct Answer :-

- $\frac{3}{4} + n^2$, $n \in \mathbb{N}$. [Option ID = 26017]

32)

Let (X, d) be a complete metric space. Then which of the following statements holds true?

[Question ID = 13976]

1. X is compact as well as connected. [Option ID = 25902]
2. If $\{F_n\}$ is a decreasing sequence of non-empty closed subsets of X then $F = \bigcap_{n=1}^{\infty} F_n$ is non-empty.

   [Option ID = 25903]
3. Every open subspace of X is complete. [Option ID = 25904]
4. If X is union of a sequence of its subsets then the closure of at least one set in the sequence must have non-empty interior.

   [Option ID = 25901]

Correct Answer :-

- If X is union of a sequence of its subsets then the closure of at least one set in the sequence must have non-empty interior.

  [Option ID = 25901]

33)

Let V be the set of all polynomials over $\mathbb{R}$. A linear transformation $D : V \to V$ is defined by $D(f(x)) = \dfrac{d^3}{dx^3}(f(x))$. Then

[Question ID = 13993]

1. dimension of kernel of D is 2. [Option ID = 25969]
2. dimension of kernel of D is 4. [Option ID = 25970]
3. range of $D = V$. [Option ID = 25972]
4. range of D is a finite dimensional space [Option ID = 25971]

Correct Answer :-

- dimension of kernel of D is 2. [Option ID = 25969]

34) If $G = \mathbb{Z}_6 \oplus \mathbb{Z}_{20} \oplus \mathbb{Z}_{72}$, then G is isomorphic to

[Question ID = 14000]

1. $\mathbb{Z}_8 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{40}$. [Option ID = 25998]
2. $\mathbb{Z}_2 \oplus \mathbb{Z}_{12} \oplus \mathbb{Z}_{360}$. [Option ID = 26000]
3. $\mathbb{Z}_5 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{64}$. [Option ID = 25997]
4. $\mathbb{Z}_6 \oplus \mathbb{Z}_{32} \oplus \mathbb{Z}_{45}$. [Option ID = 25999]

Correct Answer :-

- $\mathbb{Z}_5 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{64}$. [Option ID = 25997]

35)

The general solution of the partial differential equation

$$\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} - z = xy$$

is

[Question ID = 14006]

1. $e^x f_1(y) + e^{-y} f_2(x) + xy + y - x - 1.$ [Option ID = 26023]
2. $e^x f_1(y) + e^{-y} f_2(x) - xy - y + x + 1.$ [Option ID = 26022]
3. $e^{-x} f_1(y) + e^{y} f_2(x) + xy + y - x - 1.$ [Option ID = 26024]
4. $e^{-x} f_1(y) + e^{y} f_2(x) - xy - y + x + 1.$ [Option ID = 26021]

Correct Answer :-

- $e^{-x} f_1(y) + e^{y} f_2(x) - xy - y + x + 1.$ [Option ID = 26021]

36) The function $f : [0, 2\pi] \to S^1$ defined by $f(t) = e^{it}$, where S^1 is the unit circle, is

[Question ID = 13972]

1. continuous, one-one but not onto. [Option ID = 25886]
2. not a continuous map. [Option ID = 25885]
3. a continuous bijection but not an open map. [Option ID = 25887]
4. a homeomorphism. [Option ID = 25888]

Correct Answer :-

- not a continuous map. [Option ID = 25885]

37) Define f on $\mathbb{C}$ by

$$f(z) = \begin{cases} \frac{(\overline{z})^2}{z}, & \text{if } z \neq 0 \\ 0, & z = 0. \end{cases}$$

Let u and v denote the real and imaginary parts of f. Then at the origin

[Question ID = 13990]

1. u, v do not satisfy the Cauchy Riemann equations but f is differentiable. [Option ID = 25959]
2. u, v satisfy the Cauchy Riemann equations but f is not differentiable [Option ID = 25958]
3. f is differentiable and u, v satisfy the Cauchy Riemann equations. [Option ID = 25957]
4. f is not differentiable and u, v do not satisfy the Cauchy Riemann equations. [Option ID = 25960]

Correct Answer :-

- f is differentiable and u, v satisfy the Cauchy Riemann equations. [Option ID = 25957]

38)
Let V be the set of all polynomials over $\mathbb{R}$. Define $W = \{x^n f(x) : f(x) \in V\}$, $n \in \mathbb{N}$ is fixed. Then which of the following statements is not true?

[Question ID = 13992]

1. V is infinite dimensional over $\mathbb{R}$. [Option ID = 25967]
2. The quotient space V/W is finite dimensional. [Option ID = 25966]
3. W is not a subspace of V. [Option ID = 25965]
4. V has linearly independent set of m vectors for every $m \in \mathbb{N}$. [Option ID = 25968]

Correct Answer :-

- W is not a subspace of V. [Option ID = 25965]

39)
Navier Stokes equation of motion for steady viscous incompressible fluid flow in absence of body force is (where $\overline{q}$, p, ρ, $\overline{\varsigma}$ and ν are velocity, pressure, density, vorticity, and kinematic coefficient of viscosity respectively)

[Question ID = 14004]

1. $\nabla(\frac{1}{2}\overline{q}^2 - \frac{p}{\rho}) + \overline{q} \times \overline{\varsigma} = \nu\nabla^2\overline{q}.$ [Option ID = 26015]
2. $\nabla(\frac{1}{2}\overline{q}^2 + \frac{p}{\rho}) - \overline{q} \times \overline{\varsigma} = \nu\nabla^2\overline{q}.$ [Option ID = 26014]
3. $\nabla(\frac{1}{2}\overline{q}^2 + \frac{p}{\rho}) + \overline{q} \times \overline{\varsigma} = \nu\nabla^2\overline{q}.$ [Option ID = 26013]
4. $\nabla(\overline{q}^2 + \frac{p}{\rho}) - \overline{q} \times \overline{\varsigma} = -\nu\nabla^2\overline{q}.$ [Option ID = 26016]

Correct Answer :-

- $\nabla(\frac{1}{2}\overline{q}^2 + \frac{p}{\rho}) + \overline{q} \times \overline{\varsigma} = \nu\nabla^2\overline{q}.$ [Option ID = 26013]

40)
Let $X = C_{00}$ (the space of all real sequences having only finitely many non-zero terms) with $||\,.\,||_\infty$-norm. Define $P : X \to X$ by

$$P(x)(2j-1) = x(2j-1) + jx(2j)$$
$$P(x)(2j) = 0$$

for $x \in X$, $j \in \mathbb{N}$. Then which of the following statements is not true?

[Question ID = 13980]

1. P is closed map. [Option ID = 25918]
2. P is linear and $P^2 = P$. [Option ID = 25917]
3. Range(P) is a closed subspace of X. [Option ID = 25919]
4. P is a continuous map. [Option ID = 25920]

Correct Answer :-

- P is linear and $P^2 = P$. [Option ID = 25917]

41)

The value of $\int_C 2x\, ds$, where C consists of the arc C_1 of the parabola $y = x^2$ from $(0,\, 0)$ to $(1,\, 1)$ followed by the line segment from $(1,\, 1)$ to $(0,\, 0)$ is

[Question ID = 13971]

1. $\dfrac{5\sqrt{5}-1}{6} + 2\sqrt{2}.$ [Option ID = 25882]
2. $\dfrac{5\sqrt{5}-4}{3} + 2\sqrt{2}.$ [Option ID = 25884]
3. $\dfrac{5\sqrt{5}-1}{6} + \sqrt{2}.$ [Option ID = 25881]
4. $\dfrac{3\sqrt{5}-1}{5} + \sqrt{2}.$ [Option ID = 25883]

Correct Answer :-

- $\dfrac{5\sqrt{5}-1}{6} + \sqrt{2}.$ [Option ID = 25881]

42)

For each integer n, define $f_n(x) = x + n$, $x \in \mathbb{R}$ and let $G = \{f_n : n \in \mathbb{Z}\}$. Then

[Question ID = 13999]

1. G is a cyclic group under composition. [Option ID = 25994]
2. G is a non-cyclic group under composition. [Option ID = 25995]
3. G does not form a group under composition. [Option ID = 25993]
4. G is a non-abelian group under composition. [Option ID = 25996]

Correct Answer :-

- G does not form a group under composition. [Option ID = 25993]

43)

Suppose G is an open connected subset of $\mathbb{C}$ containing 0 and $f : G \to \mathbb{C}$ is analytic such that $f(0) = 0$ and $|f(z) - 1| = 1$ for all $z \in G$. Then the range of f is

1. $\{0, 2\}$. [Option ID = 25954]
2. $\{1 + e^{i\theta} : 0 \leq \theta \leq \pi\}$ [Option ID = 25956]
3. $\{1 + e^{i\theta} : 0 \leq \theta \leq 2\pi\}$. [Option ID = 25953]
4. $\{0\}$ [Option ID = 25955]

Correct Answer :-

- $\{1 + e^{i\theta} : 0 \leq \theta \leq 2\pi\}$. [Option ID = 25953]

44) Consider the following statements:
Dimension of kinematic coefficient of viscosity is
(I) L^2T^{-1}.
(II) same as dimension of stream function.
(III) $L^{-2}T^1$.
(IV) same as dimension of stokes stream function.
Then

[Question ID = 14003]

1. Only (III) and (IV) are true. [Option ID = 26012]
2. Only (I) and (II) are true. [Option ID = 26009]
3. Only (II) and (III) are true. [Option ID = 26011]
4. Only (I) and (IV) are true. [Option ID = 26010]

Correct Answer :-

- Only (I) and (II) are true. [Option ID = 26009]

45)

Consider a sequence $\{x_n\}$ defined by $0 < x_1 < 1$ and $x_{n+1} = 1 - \sqrt{1 - x_n}$, $n = 1, 2, \cdots$. Then $\frac{x_{n+1}}{x_n}$ converges to

[Question ID = 13967]

1. 0 [Option ID = 25866]
2. 1/3 [Option ID = 25867]
3. 1/2 [Option ID = 25868]
4. 1 [Option ID = 25865]

Correct Answer :-

- 1 [Option ID = 25865]

46)
Which of the following statements about the outer measure m^* on $\mathbb{R}$ is true?

[Question ID = 13987]

1. There exists an open subset $A \subseteq \mathbb{R}$ such that $m^*A = 0$. [Option ID = 25945]
2. Every subset of $\mathbb{R}$ of zero outer measure is at most countable. [Option ID = 25947]
3. If $B \subseteq \mathbb{R}$ is unbounded, then $m^*B > 0$. [Option ID = 25948]
4. Every non empty closed subset E of $\mathbb{R}$ has $m^*E > 0$. [Option ID = 25946]

Correct Answer :-

- There exists an open subset $A \subseteq \mathbb{R}$ such that $m^*A = 0$. [Option ID = 25945]

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47) Which of the following statements is true? [Question ID = 13977]

1. In a metric space, the image of a Cauchy sequence under a continuous map is a Cauchy sequence . [Option ID = 25906]
2. Every closed and bounded subset of a metric space is compact. [Option ID = 25907]
3. Every infinite subset of the closed unit ball B in $\mathbb{R}^n$ has a limit point in B. [Option ID = 25905]
4. In a metric space, every closed ball of positive radius is connected. [Option ID = 25908]

Correct Answer :-

- Every infinite subset of the closed unit ball B in $\mathbb{R}^n$ has a limit point in B. [Option ID = 25905]

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48) Which one of the following statements is not true? [Question ID = 13966]

1. There is a function f defined on $\mathbb{R}$ which is continuous on $\mathbb{Q}$ (rational numbers) and discontinuous on $\mathbb{Q}'$(irrational numbers). [Option ID = 25861]
2. Monotone convergence property is equivalent to completeness of $\mathbb{R}$. [Option ID = 25864]
3. Bolzano-Weiestrass theorem is equivalent to completeness of $\mathbb{R}$. [Option ID = 25863]
4. Cantor's intersection property of $\mathbb{R}$ is equivalent to completeness of $\mathbb{R}$ [Option ID = 25862]

Correct Answer :-

- There is a function f defined on $\mathbb{R}$ which is continuous on $\mathbb{Q}$ (rational numbers) and discontinuous on $\mathbb{Q}'$(irrational numbers). [Option ID = 25861]

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49) Present President of the Ramanujan Mathematical Society is [Question ID = 13916]

1. V. Kumar Murty. [Option ID = 25664]

2. Dinesh Singh [Option ID = 25661]
3. S. Ponnusamy [Option ID = 25662]
4. R. Balakrishnan. [Option ID = 25663]

Correct Answer :-

- Dinesh Singh [Option ID = 25661]

50) The characteristic and the minimal polynomial are same for the matrix

[Question ID = 13996]

1. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ [Option ID = 25983]
2. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ [Option ID = 25982]
3. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ [Option ID = 25981]
4. All of the above matrices [Option ID = 25984]

Correct Answer :-

- $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ [Option ID = 25981]