Q. 1 – Q. 25 carry one mark each.

Q.1 A satellite is moving in a circular orbit around the Earth. If T, V and E are its average kinetic, average potential and total energies, respectively, then which one of the following options is correct?

(A) $V = -2T;\ E = -T$
(B) $V = -T;\ E = 0$
(C) $V = -T/2;\ E = T/2$
(D) $V = -3T/2;\ E = -T/2$

Q.2 The Pauli matrices for three spin-½ particles are $\vec{\sigma}_1$, $\vec{\sigma}_2$, and $\vec{\sigma}_3$, respectively. The dimension of the Hilbert space required to define an operator $\hat{O} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 \times \vec{\sigma}_3$ is __________

Q.3 The mean kinetic energy of a nucleon in a nucleus of atomic weight A varies as A^n, where n is ______________ (upto two decimal places)

Q.4 Let $\vec{L}$ and $\vec{p}$ be the angular and linear momentum operators, respectively, for a particle. The commutator $[L_x, p_y]$ gives

(A) $-i\hbar\, p_z$
(B) 0
(C) $i\hbar\, p_x$
(D) $i\hbar\, p_z$

Q.5 The decay $\mu^+ \rightarrow e^+ + \gamma$ is forbidden, because it violates

(A) momentum and lepton number conservations
(B) baryon and lepton number conservations
(C) angular momentum conservation
(D) lepton number conservation

Q.6 An operator for a spin-½ particle is given by $\hat{A} = \lambda \vec{\sigma} \cdot \vec{B}$, where $\vec{B} = \frac{B}{\sqrt{2}}(\hat{x} + \hat{y})$, $\vec{\sigma}$ denotes Pauli matrices and λ is a constant. The eigenvalues of $\hat{A}$ are

(A) $\pm \lambda B/\sqrt{2}$
(B) $\pm \lambda B$
(C) $0,\ \lambda B$
(D) $0,\ -\lambda B$

Q.7 In an inertial frame S, two events A and B take place at $(ct_A = 0,\ \vec{r}_A = 0)$ and $(ct_B = 0,\ \vec{r}_B = 2\hat{y})$, respectively. The times at which these events take place in a frame S' moving with a velocity $0.6c\,\hat{y}$ with respect to S are given by

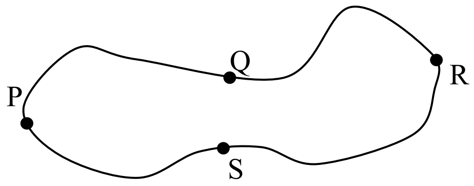
(A) $ct'_A = 0; ct'_B = -3/2$

(B) $ct'_A = 0; ct'_B = 0$

(C) $ct'_A = 0; ct'_B = 3/2$

(D) $ct'_A = 0; ct'_B = 1/2$

Q.8 Given that the magnetic flux through the closed loop PQRSP is ϕ. If $\int_P^R \vec{A}\cdot\vec{dl} = \phi_1$ along PQR, the value of $\int_P^R \vec{A}\cdot\vec{dl}$ along PSR is



(A) $\phi - \phi_1$

(B) $\phi_1 - \phi$

(C) $-\phi_1$

(D) ϕ_1

Q.9 If $f(x) = e^{-x^2}$ and $g(x) = |x|e^{-x^2}$, then

(A) f and g are differentiable everywhere

(B) f is differentiable everywhere but g is not

(C) g is differentiable everywhere but f is not

(D) g is discontinuous at $x = 0$

Q.10 In Bose-Einstein condensates, the particles

(A) have strong interparticle attraction

(B) condense in real space

(C) have overlapping wavefuntions

(D) have large and positive chemical potential

Q.11 Consider a system of N non-interacting spin-½ particles, each having a magnetic moment μ, is in a magnetic field $\vec{B} = B\,\hat{z}$. If E is the total energy of the system, the number of accessible microstates Ω is given by

(A) $\Omega = \dfrac{N!}{\frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\ \frac{1}{2}\left(N + \frac{E}{\mu B}\right)!}$

(B) $\Omega = \dfrac{\left(N - \frac{E}{\mu B}\right)!}{\left(N + \frac{E}{\mu B}\right)!}$

(C) $\Omega = \frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\ \frac{1}{2}\left(N + \frac{E}{\mu B}\right)!$

(D) $\Omega = \dfrac{N!}{\left(N + \frac{E}{\mu B}\right)!}$

Q.12 For a black body radiation in a cavity, photons are created and annihilated freely as a result of emission and absorption by the walls of the cavity. This is because

(A) the chemical potential of the photons is zero

(B) photons obey Pauli exclusion principle

(C) photons are spin-1 particles

(D) the entropy of the photons is very large

Q.13 Consider $w = f(z) = u(x, y) + iv(x, y)$ to be an analytic function in a domain D. Which one of the following options is **NOT** correct?

(A) $u(x, y)$ satisfies Laplace equation in D

(B) $v(x, y)$ satisfies Laplace equation in D

(C) $\int_{z_1}^{z_2} f(z)\,dz$ is dependent on the choice of the contour between z_1 and z_2 in D

(D) $f(z)$ can be Taylor expanded in D

Q.14 The value of $\int_0^3 t^2\,\delta(3t-6)\,dt$ is ________________ (upto one decimal place)

Q.15 Which one of the following **DOES NOT** represent an exclusive OR operation for inputs A and B ?

(A) $(A+B)\overline{AB}$ (B) $A\bar{B}+B\bar{A}$

(C) $(A+B)(\bar{A}+\bar{B})$ (D) $(A+B)AB$

Q.16 Consider a complex function $f(z)=\frac{1}{z\left(z+\frac{1}{2}\right)\cos(z\pi)}$. Which one of the following statements is correct?

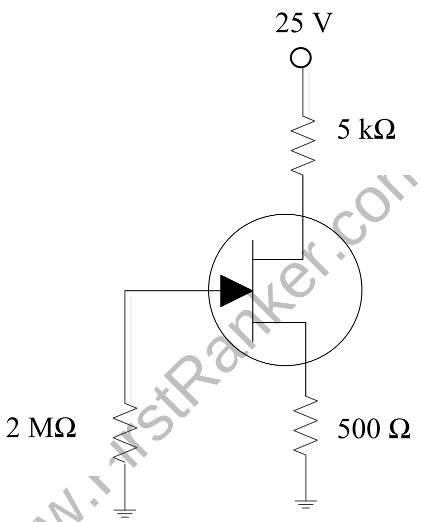
(A) $f(z)$ has simple poles at $z=0$ and $z=-1/2$

(B) $f(z)$ has a second order pole at $z=-1/2$

(C) $f(z)$ has infinite number of second order poles

(D) $f(z)$ has all simple poles

Q.17 In the given circuit, the voltage across the source resistor is 1 V. The drain voltage (in V) is _______



Q.18 A point charge is placed between two semi-infinite conducting plates which are inclined at an angle of 30° with respect to each other. The number of image charges is ______________

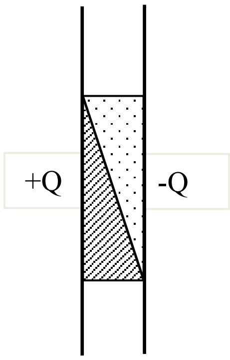
Q.19 A beam of X-ray of intensity I_0 is incident normally on a metal sheet of thickness 2 *mm*. The intensity of the transmitted beam is $0.025I_0$. The linear absorption coefficient of the metal sheet (in m^{-1}) is _______________ (upto one decimal place)

Q.20 The lattice parameters a, b, c of an orthorhombic crystal are related by $a = 2b = 3c$. In units of a, the interplanar separation between the (110) planes is __________ (upto three decimal places)

Q.21 In a Hall effect experiment, the Hall voltage for an intrinsic semiconductor is negative. This is because (symbols carry usual meaning)

(A) $n \approx p$ (B) $n > p$ (C) $\mu_e > \mu_h$ (D) $m_e^* > m_h^*$

Q.22 The space between two plates of a capacitor carrying charges +Q and –Q is filled with two different dielectric materials, as shown in the figure. Across the interface of the two dielectric materials, which one of the following statements is correct?



(A) $\vec{E}$ and $\vec{D}$ are continuous

(B) $\vec{E}$ is continuous and $\vec{D}$ is disconitnuous

(C) $\vec{D}$ is continuous and $\vec{E}$ is discontinuous

(D) $\vec{E}$ and $\vec{D}$ are discontinuous

Q.23 The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by, $g(E) = CE^n$, where C is constant. The value of n is ______________

Q.24 The dispersion relation for phonons in a one dimensional monatomic Bravais lattice with lattice spacing a and consisting of ions of masses M is given by, $\omega(k) = \sqrt{\frac{2C}{M}\left[1-\cos(ka)\right]}$, where ω is the frequency of oscillation, k is the wavevector and C is the spring constant. For the long wavelength modes $(\lambda >> a)$, the ratio of the phase velocity to the group velocity is _________

Q.25 Four forces are given below in Cartesian and spherical polar coordinates.

(i) $\vec{F}_1 = K\exp(-r^2/R^2)\hat{r}$

(ii) $\vec{F}_2 = K(x^3\hat{y} - y^3\hat{z})$

(iii) $\vec{F}_3 = K(x^3\hat{x} + y^3\hat{y})$

(iv) $\vec{F}_4 = K(\hat{\phi}/r)$

where K is a constant. Identify the correct option.

(A) (iii) and (iv) are conservative but (i) and (ii) are not

(B) (i) and (ii) are conservative but (iii) and (iv) are not

(C) (ii) and (iii) are conservative but (i) and (iv) are not

(D) (i) and (iii) are conservative but (ii) and (iv) are not

Q. 26 – Q. 55 carry two marks each.

Q.26 Consider a system of eight non-interacting, identical quantum particles of spin-3/2 in a one dimensional box of length L. The minimum excitation energy of the system, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is ___________

Q.27 The excitation wavelength of laser in a Raman effect experiment is 546 nm. If the Stokes' line is observed at 552 nm, then the wavenumber of the anti-Stokes' line (in cm^{-1}) is ___________

Q.28 The binding energy per molecule of NaCl (lattice parameter is 0.563 nm) is 7.95 eV. The repulsive term of the potential is of the form $\frac{K}{r^9}$, where K is a constant. The value of the Madelung constant is ____________ (upto three decimal places)

(Electron charge $e = -1.6 \times 10^{-19}\ C$; $\varepsilon_0 = 8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}$)

Q.29 Given that the Fermi energy of gold is 5.54 eV, the number density of electrons is ____________ $\times 10^{28}\ m^{-3}$ (upto one decimal place)

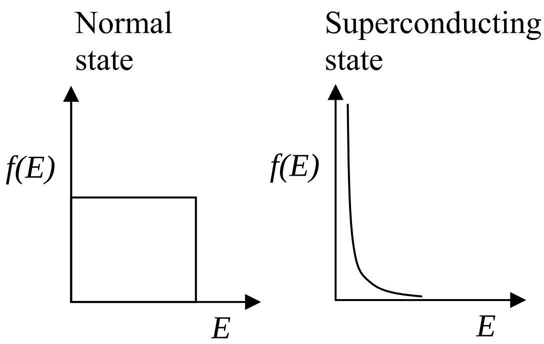
(Mass of electron = $9.11 \times 10^{-31}\ kg$; $h = 6.626 \times 10^{-34} J.s$; $1\ eV = 1.6 \times 10^{-19} J$)

Q.30 The band gap of an intrinsic semiconductor is $E_g = 0.72\ eV$ and $m_h^* = 6m_e^*$. At 300 K, the Fermi level with respect to the edge of the valence band (in eV) is at ________ (upto three decimal places) $k_B = 1.38 \times 10^{-23} JK^{-1}$

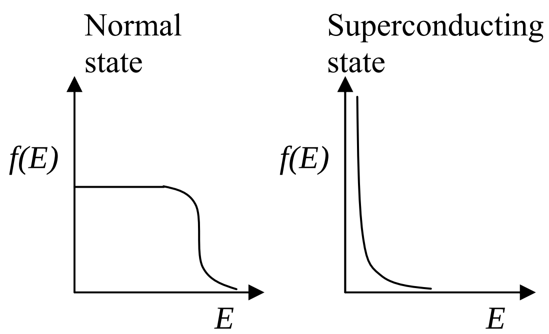
Q.31 The number of permitted transitions from $^2P_{3/2} \rightarrow {}^2S_{1/2}$ in the presence of a weak magnetic field is ________________

Q.32 Which one of the following represents the electron occupancy for a superconductor in its normal and superconducting states?

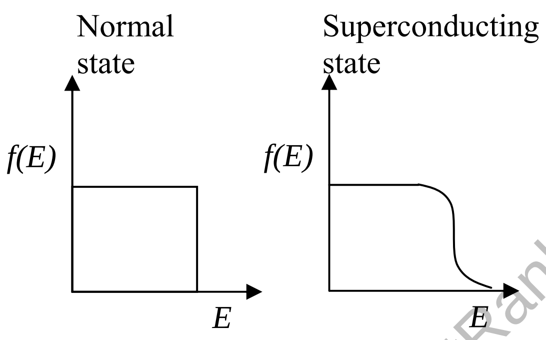
(A)



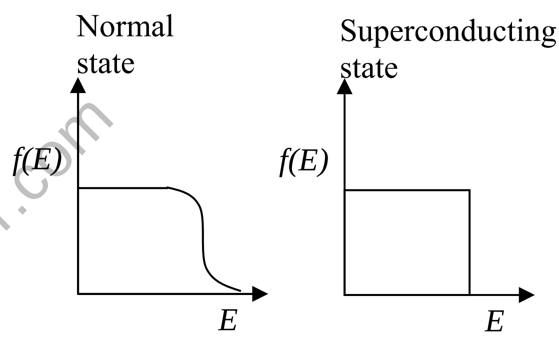
(B)



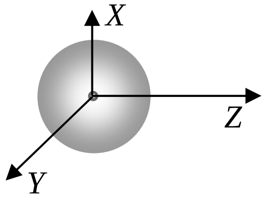
(C)



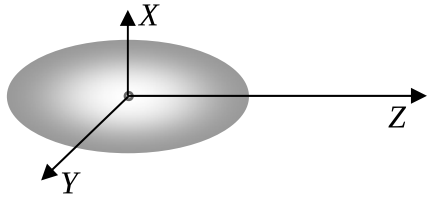
(D)



Q.33 A charge $-q$ is distributed uniformly over a sphere, with a positive charge q at its center in (i). Also in (ii), a charge $-q$ is distributed uniformly over an ellipsoid with a positive charge q at its center. With respect to the origin of the coordinate system, which one of the following statements is correct?



(i)



(ii)

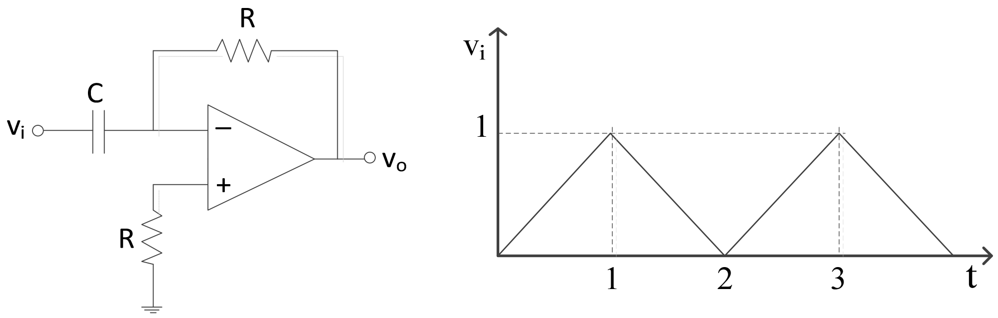
(A) The dipole moment is zero in both (i) and (ii)

(B) The dipole moment is non-zero in (i) but zero in (ii)

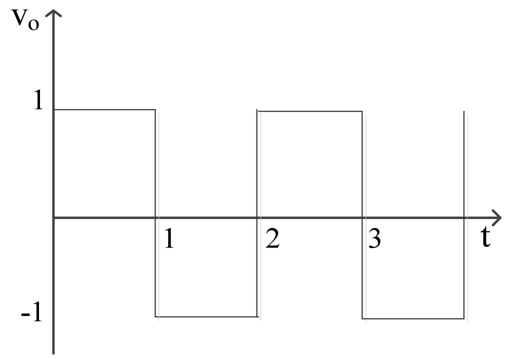
(C) The dipole moment is zero in (i) but non-zero in (ii)

(D) The dipole moment is non-zero in both (i) and (ii)

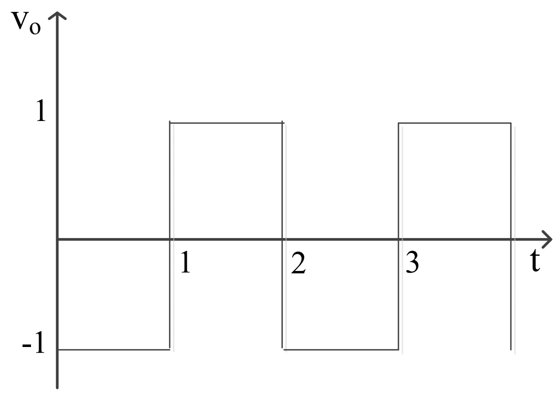
Q.34 Consider the circuit shown in the figure, where RC = 1. For an input signal V_i shown below, choose the correct V_o from the options:



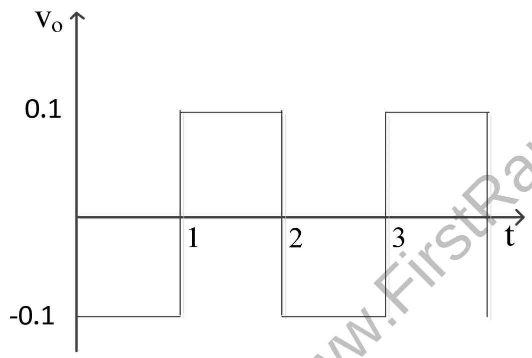
(A)



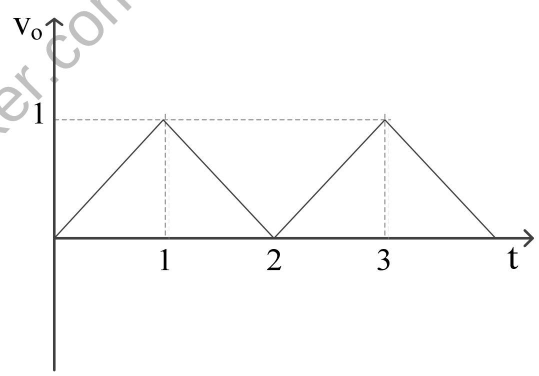
(B)



(C)



(D)



Q.35 A long solenoid is embedded in a conducting medium and is insulated from the medium. If the current through the solenoid is increased at a constant rate, the induced current in the medium as a function of the radial distance r from the axis of the solenoid is proportional to

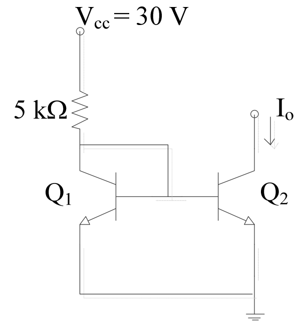
(A) r^2 inside the solenoid and $\frac{1}{r}$ outside

(B) r inside the solenoid and $\frac{1}{r^2}$ outside

(C) r^2 inside the solenoid and $\frac{1}{r^2}$ outside

(D) r inside the solenoid and $\frac{1}{r}$ outside

Q.36 In the *simple current source* shown in the figure, Q_1 and Q_2 are identical transistors with current gain $\beta = 100$ and $V_{BE} = 0.7\ V$



The current I_o (in *mA*) is ___________ (upto two decimal places)

Q.37 Match the phrases in Group I and Group II and identify the correct option.

Group I	Group II
(P) Electron spin resonance (ESR)	(i) radio frequency
(Q) Nuclear magnetic resonance (NMR)	(ii) visible range frequency
(R) Transition between vibrational states of a molecule	(iii) microwave frequency
(S) Electronic transition	(iv) far-infrared range

(A) (P-i), (Q-ii), (R-iii), (S-iv)
(B) (P-ii), (Q-i), (R-iv), (S-iii)
(C) (P-iii), (Q-iv), (R-i), (S-ii)
(D) (P-iii), (Q-i), (R-iv), (S-ii)

Q.38 Consider the motion of the Sun with respect to the rotation of the Earth about its axis. If $\vec{F}_c$ and $\vec{F}_{Co}$ denote the centrifugal and the Coriolis forces, respectively, acting on the Sun, then

(A) $\vec{F}_c$ is radially outward and $\vec{F}_{Co} = \vec{F}_c$

(B) $\vec{F}_c$ is radially inward and $\vec{F}_{Co} = -2\vec{F}_c$

(C) $\vec{F}_c$ is radially outward and $\vec{F}_{Co} = -2\vec{F}_c$

(D) $\vec{F}_c$ is radially outward and $\vec{F}_{Co} = 2\vec{F}_c$

Q.39 In a rigid-rotator of mass *M*, if the energy of the first excited state is 1 *meV*, then the fourth excited state energy (in *meV*) is _____________

Q.40 A plane wave $(\hat{x}+i\hat{y})E_0 \exp[i(kz-\omega t)]$ after passing through an optical element emerges as $(\hat{x}-i\hat{y})E_0 \exp[i(kz-\omega t)]$, where k and ω are the wavevector and the angular frequency, respectively. The optical element is a

(A) quarter wave plate (B) half wave plate

(C) polarizer (D) Faraday rotator

Q.41 The Lagrangian for a particle of mass m at a position $\vec{r}$ moving with a velocity $\vec{v}$ is given by $L = \frac{m}{2}\vec{v}^2 + C\,\vec{r}.\vec{v} - V(r)$, where $V(r)$ is a potential and C is a constant. If $\vec{p}_c$ is the canonical momentum, then its Hamiltonian is given by

(A) $\frac{1}{2m}(\vec{p}_c + C\,\vec{r})^2 + V(r)$ (B) $\frac{1}{2m}(\vec{p}_c - C\,\vec{r})^2 + V(r)$

(C) $\frac{p_c^2}{2m} + V(r)$ (D) $\frac{1}{2m}p_c^2 + C^2 r^2 + V(r)$

Q.42 The Hamiltonian for a system of two particles of masses m_1 and m_2 at $\vec{r}_1$ and $\vec{r}_2$ having velocities $\vec{v}_1$ and $\vec{v}_2$ is given by $H = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{C}{(\vec{r}_1 - \vec{r}_2)^2}\hat{z}\cdot(\vec{r}_1 \times \vec{r}_2)$, where C is a constant. Which one of the following statements is correct?

(A) The total energy and total momentum are conserved

(B) Only the total energy is conserved

(C) The total energy and the z - component of the total angular momentum are conserved

(D) The total energy and total angular momentum are conserved

Q.43 A particle of mass 0.01 kg falls freely in the earth's gravitational field with an initial velocity $v(0) = 10\,\text{ms}^{-1}$. If the air exerts a frictional force of the form, $f = -kv$, then for $k = 0.05\ Nm^{-1}s$, the velocity (in ms^{-1}) at time $t = 0.2\ s$ is ________ (upto two decimal places)

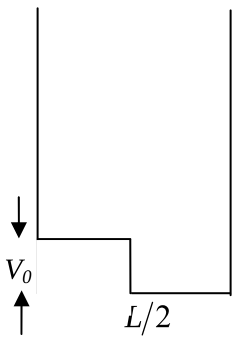
(use $g = 10\ ms^{-2}$ and $e = 2.72$)

Q.44 In the nuclear shell model, the potential is modeled as $V(r) = \frac{1}{2} m\omega^2 r^2 - \lambda \vec{L}\cdot\vec{S}, \quad \lambda > 0$. The correct spin - parity and isospin assignments for the ground state of ^{13}C is

(A) $\frac{1}{2}^{-}; \frac{-1}{2}$ (B) $\frac{1}{2}^{+}; \frac{-1}{2}$

(C) $\frac{3}{2}^{+}; \frac{1}{2}$ (D) $\frac{3}{2}^{-}; \frac{-1}{2}$

Q.45 A particle is confined in a box of length L as shown below.



If the potential V_0 is treated as a perturbation, including the first order correction, the ground state energy is

(A) $E = \frac{\hbar^2\pi^2}{2mL^2} + V_0$ (B) $E = \frac{\hbar^2\pi^2}{2mL^2} - \frac{V_0}{2}$

(C) $E = \frac{\hbar^2\pi^2}{2mL^2} + \frac{V_0}{4}$ (D) $E = \frac{\hbar^2\pi^2}{2mL^2} + \frac{V_0}{2}$

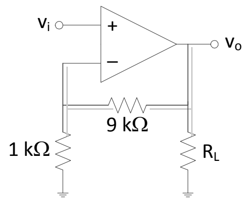
Q.46 Suppose a linear harmonic oscillator of frequency ω and mass m is in the state $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\psi_0\rangle + e^{i\frac{\pi}{2}}|\psi_1\rangle\right]$ at $t = 0$ where $|\psi_0\rangle$ and $|\psi_1\rangle$ are the ground and the first excited states, respectively. The value of $\langle\psi|x|\psi\rangle$ in the units of $\sqrt{\frac{\hbar}{m\omega}}$ at $t = 0$ is ___________

Q.47 A particle with rest mass M is at rest and decays into two particles of equal rest masses $\frac{3}{10}M$ which move along the z axis. Their velocities are given by

(A) $\vec{v}_1 = \vec{v}_2 = (0.8c)\hat{z}$ (B) $\vec{v}_1 = -\vec{v}_2 = (0.8c)\hat{z}$

(C) $\vec{v}_1 = -\vec{v}_2 = (0.6c)\hat{z}$ (D) $\vec{v}_1 = (0.6c)\hat{z}; \quad \vec{v}_2 = (-0.8c)\hat{z}$

Q.48 In the given circuit, if the open loop gain $A = 10^5$, the feedback configuration and the closed loop gain A_f are



(A) series-shunt, $A_f = 9$
(B) series-series, $A_f = 10$
(C) series-shunt, $A_f = 10$
(D) shunt-shunt, $A_f = 10$

Q.49 A function $y(z)$ satisfies the ordinary differential equation $y'' + \frac{1}{z}y' - \frac{m^2}{z^2}y = 0$, where $m = 0, 1, 2, 3, \ldots$. Consider the four statements P, Q, R, S as given below.

P: z^m and z^{-m} are linearly independent solutions for all values of m
Q: z^m and z^{-m} are linearly independent solutions for all values of $m>0$
R: $\ln z$ and 1 are linearly independent solutions for $m = 0$
S: z^m and $\ln z$ are linearly independent solutions for all values of m

The correct option for the combination of valid statements is

(A) P, R and S only (B) P and R only (C) Q and R only (D) R and S only

Q.50 The entropy of a gas containing N particles enclosed in a volume V is given by $S = Nk_B \ln\left(\frac{aVE^{3/2}}{N^{5/2}}\right)$, where E is the total energy, a is a constant and k_B is the Boltzmann constant. The chemical potential μ of the system at a temperature T is given by

(A) $\mu = -k_B T\left[\ln\left(\frac{aVE^{3/2}}{N^{5/2}}\right) - \frac{5}{2}\right]$
(B) $\mu = -k_B T\left[\ln\left(\frac{aVE^{3/2}}{N^{5/2}}\right) - \frac{3}{2}\right]$
(C) $\mu = -k_B T\left[\ln\left(\frac{aVE^{3/2}}{N^{3/2}}\right) - \frac{5}{2}\right]$
(D) $\mu = -k_B T\left[\ln\left(\frac{aVE^{3/2}}{N^{3/2}}\right) - \frac{3}{2}\right]$

Q.51 Let the Hamiltonian for two spin-½ particles of equal masses m, momenta $\vec{p}_1$ and $\vec{p}_2$ and positions $\vec{r}_1$ and $\vec{r}_2$ be $H = \frac{1}{2m}p_1^2 + \frac{1}{2m}p_2^2 + \frac{1}{2}m\omega^2\left(r_1^2 + r_2^2\right) + k\vec{\sigma}_1 \cdot \vec{\sigma}_2$, where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ denote the corresponding Pauli matrices, $\hbar\omega = 0.1\,eV$ and $k = 0.2\,eV$. If the ground state has net spin zero, then the energy (in eV) is ___________

Q.52 The average energy U of a one dimensional quantum oscillator of frequency ω and in contact with a heat bath at temperature T is given by

(A) $U = \frac{1}{2}\hbar\omega\coth\left(\frac{1}{2}\beta\hbar\omega\right)$

(B) $U = \frac{1}{2}\hbar\omega\sinh\left(\frac{1}{2}\beta\hbar\omega\right)$

(C) $U = \frac{1}{2}\hbar\omega\tanh\left(\frac{1}{2}\beta\hbar\omega\right)$

(D) $U = \frac{1}{2}\hbar\omega\cosh\left(\frac{1}{2}\beta\hbar\omega\right)$

Q.53 A monochromatic plane wave (wavelength = 600 nm) $E_0\exp[i(kz - \omega t)]$ is incident normally on a diffraction grating giving rise to a plane wave $E_1\exp\left[i\left(\vec{k}_1\cdot\vec{r} - \omega t\right)\right]$ in the first order of diffraction. Here $E_1 < E_0$ and $\vec{k}_1 = \left|\vec{k}_1\right|\left[\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{z}\right]$. The period (in μm) of the diffraction grating is _____________ (upto one decimal place)

Q.54 The Heaviside function is defined as $H(t) = \begin{cases} +1 & \text{for } t > 0 \\ -1 & \text{for } t < 0 \end{cases}$ and its Fourier transform is given by $-2i/\omega$. The Fourier transform of $\frac{1}{2}[H(t+1/2) - H(t-1/2)]$ is

(A) $\dfrac{\sin\left(\frac{\omega}{2}\right)}{\omega/2}$

(B) $\dfrac{\cos\left(\frac{\omega}{2}\right)}{\omega/2}$

(C) $\sin\left(\frac{\omega}{2}\right)$

(D) 0

Q.55 The atomic masses of ${}^{152}_{63}Eu$, ${}^{152}_{62}Sm$, ${}^{1}_{1}H$ and neutron are 151.921749, 151.919756, 1.007825 and 1.008665 in atomic mass units (amu), respectively. Using the above information, the Q-value of the reaction ${}^{152}_{63}Eu + n \rightarrow {}^{152}_{62}Sm + p$ is __________ $\times 10^{-3}$ amu (upto three decimal places)

END OF THE QUESTION PAPER