

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER- III (New) EXAMINATION – WINTER 2019
Subject Code: 3130005
Date: 26/11/2019
Subject Name: Complex Variables and Partial Differential Equations
Time: 02:30 PM TO 05:00 PM
Total Marks: 70
Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Marks

- Q.1**
- (a) Find the real and imaginary parts of $f(z) = \frac{3i}{2+3i}$. **03**
- (b) State De-Moivre's formula and hence evaluate **04**
 $(1+i\sqrt{3})^{100} + (1-i\sqrt{3})^{100}$.
- (c) Define harmonic function. Show that $u(x, y) = \sinh x \sin y$ is harmonic **07**
 function, find its harmonic conjugate $v(x, y)$.
- Q.2**
- (a) Determine the Mobius transformation which maps $z_1 = 0, z_2 = 1, z_3 = \infty$ **03**
 into $w_1 = -1, w_2 = -i, w_3 = 1$.
- (b) Define $\log z$, prove that $i^i = e^{-(4n+1)\frac{\pi}{2}}$. **04**
- (c) Expand $f(z) = \frac{1}{(z-1)(z+2)}$ valid for the region **07**
 (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$.
- OR**
- (c) Find the image of the infinite strips (i) $\frac{1}{4} \leq y \leq \frac{1}{2}$ (ii) $0 < y < \frac{1}{2}$ under the **07**
 transformation $w = \frac{1}{z}$. Show the region graphically.
- Q.3**
- (a) Evaluate $\int_c (x - y + ix^2) dz$ where c is a straight line from $z = 0$ to $z =$ **03**
 $1 + i$.
- (b) Check whether the following functions are analytic or not at any point, **04**
 (i) $f(z) = 3x + y + i(3y - x)$ (ii) $f(z) = z^{3/2}$.
- (c) Using residue theorem, evaluate $\int_0^\infty \frac{dx}{(x^2+1)^2}$. **07**
- OR**
- Q.3**
- (a) Expand Laurent series of $f(z) = \frac{1-e^z}{z}$ at $z = 0$ and identify the **03**
 singularity.
- (b) If $f(z) = u + iv$, is an analytic function, prove that **04**
 $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |Re f(z)|^2 = 2|f'(z)|^2$.
- (c) Evaluate the following: **07**
- i. $\int_c \frac{z+3}{z-1} dz$ where c is the circle (a) $|z| = 2$ (b) $|z| = \frac{1}{2}$.
- ii. $\int_c \frac{\sin z}{(z-\frac{\pi}{4})^3} dz$ where c is the circle $|z| = 1$.

- Q.4** (a) Evaluate $\int_0^{2+4i} \operatorname{Re}(z) dz$ along the curve $z(t) = t + it^2$. 03
 (b) Solve $x^2 p + y^2 q = (x + y)z$. 04
 (c) Solve the equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ for the condition of heat along rod without radiation subject to the conditions (i) $\frac{\partial u}{\partial t} = 0$ for $x = 0$ and $x = l$;
 (ii) $u = lx - x^2$ at $t = 0$ for all x . 07

OR

- Q.4** (a) Solve $\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{2x+3y}$. 03
 (b) Solve $px + qy = pq$ using Charpit's method. 04
 (c) Find the general solution of partial differential equation $u_{xx} = 9u_y$ using method of separation of variables. 07
- Q.5** (a) Using method of separation of variables, solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$. 03
 (b) Solve $z(xp - yq) = y^2 - x^2$. 04
 (c) A string of length $L = \pi$ has its ends fixed at $x = 0$ and $x = \pi$. At time $t = 0$, the string is given a shape defined by $f(x) = 50x(\pi - x)$, then it is released. Find the deflection of the string at any time t . 07

OR

- Q.5** (a) Solve $p^3 + q^3 = x + y$. 03
 (b) Find the temperature in the thin metal rod of length l with both the ends insulated and initial temperature is $\sin \frac{\pi x}{l}$. 04
 (c) Derive the one dimensional wave equation that governs small vibration of an elastic string. Also state physical assumptions that you make for the system. 07
