# GUJARAT TECHNOLOGICAL UNIVERSITY <br> BE - SEMESTER- III (New) EXAMINATION - WINTER 2019 

Subject Code: 3130006
Date: 26/11/2019
Subject Name: Probability and Statistics
Time: 02:30 PM TO 05:00 PM
Instructions:
Total Marks: 70

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
Q. 1 (a) In how many different ways can 4 of 15 laboratory assistants be chosen to assist with an experiment?
(b) If 5 of 20 tires in storage are defective and 5 of them are randomly chosen for inspection (that is, each tire has the same chance of being selected), what is the probability that the two of the defective tires will be included?
(c) The following are the data on the drying time of a certain varnish and the amount of an additive that is intended to reduce the drying time?

| Amount of varnish <br> additive(grams)" x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Drying time(hr) " $\mathrm{y} "$ | 12.0 | 10.5 | 10.0 | 8.0 | 7.0 | 8.0 | 7.5 | 8.5 | 9.0 |

(i) Fit a second degree polynomial by the method of least square.
(ii) Use the result of (i) to predict the drying time of the varnish when 6.5 gms of the additive is being used.
Q. 2 (a) If 3 balls are "randomly drawn" from a bowl containing 6 white and 5 black balls. What is the probability that one of the balls is white and the other two black?
(b) The article "A Thin-Film Oxygen Uptake Test for the Evaluation of Automotive

Crankcase Lubricants" reported the following data on oxidation-induction time (min) for various commercial oils:
$87,103,130,160,180,195,132,145,211,105,145,153,152,138,87,99,93,119$, 129
(i) Calculate the sample variance and standard deviation.
(ii) If the observations were re-expressed in hours, what would be the resulting values of the sample variance and sample standard deviation?
(c) In an examination, minimum 40 marks for passing and 75 marks for distinction are required. In this examination $45 \%$ students passed and $9 \%$ obtained distinction. Find average marks and standard deviation of this distribution of marks.
$[\mathrm{P}(\mathrm{z}=0.125)=0.05$ and $\mathrm{P}(\mathrm{z}=1.34)=0.41]$
OR
(c) Distribution of height of 1000 students is normal with mean 165 cms and standard deviation 15 cms . How many soldiers are of height
(i) less than 138 cms (ii) more than 198 cms (iii) between 138 and 198 cms .
$[\mathrm{P}(\mathrm{z}=1.8)=0.4641, \mathrm{P}(\mathrm{z}=2.2)=0.4861]$
Q. 3 (a) Compute the coefficient of correlation between X and Y using the following data:

| X | 2 | 4 | 5 | 6 | 8 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y | 18 | 12 | 10 | 8 | 7 | 5 |

(b) An analysis of monthly wages paid to workers in two firms A and B belong to the same

| Www.FirstRanker.com | FirmBW.FirstRanker.com |  |
| :--- | :--- | :--- |
| No. of wages earners | 986 | 548 |
| Average monthly wages | Rs. 52.5 | Rs. 47.5 |
| Variance of distribution of wages | 100 | 121 |

(a) Which firm pays out large amounts as wage bill?
(b) In which firm there is greater variability in individual wages?
(c) Obtain the two lines of regression for the following data:

| Sales <br> (No. of tablets) | 190 | 240 | 250 | 300 | 310 | 335 | 300 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Advertising <br> expenditure (Rs.) | 5 | 10 | 15 | 20 | 20 | 30 | 30 |

Q. 3 (a) A sample of 20 items has mean 42 units and standard deviation 5 units. Test the hypothesis that it is a random sample from a normal population with mean 45 units.[t at $5 \%$ level for 19 d.f. is 2.09.]
(b) A university warehouse has received a shipment of 25 printers, of which 10 are laser printers and 15 are inkjet models. If 6 of these 25 are selected at random to be checked by a particular technician, what is the probability that exactly 3 of those selected are laser printers (so that the other 3 are inkjets)?
(c) Find the regression equation showing the capacity utilization on production from the following data:

|  | Average | Standard deviation |
| :--- | :--- | :--- |
| Production (in lakh units) | 35.6 | 10.5 |
| Capacity utilization (in \%) | 84.8 | 8.5 |
| Correlation coefficient | $\mathrm{r}=0.62$ |  |

Estimate the production when capacity utilization is $70 \%$.
Q. 4 (a) Each sample of water has $\mathrm{a}-10 \%$ chance of containing a particular organic pollutant.

Assume that the samples are independent with regard to the presence of the pollutant.
Find the probability that in the next 18 samples, at least 4 samples contain the pollutant.
(b) Goal scored by two teams A and B in a football season were as follows:

| No. of goals scored in a match | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of matches played by team A | 27 | 9 | 8 | 5 | 4 |
| No. of matches played by team B | 17 | 9 | 6 | 5 | 3 |

Find out which team is more consistent.
(c) Out of 800 families with 4 children each, how many families would be expected to have
(i) 2 girls and 2 boys (ii) at least one boy (iii) no girl (iv) at most two girls? Assume equal probabilities for boys and girls.

> OR
Q. 4 (a) Assume that the probability that a wafer contains a large particle of contamination is 0.01 and that the wafers are independent; that is, the probability that a wafer contains a large particle is not dependent on the characteristics of any of the other wafers. If 15 wafers are analyzed, what is the probability that no large particles are found?
 of the chips, but $5 \%$ of its chips are defective. Machine II produces $35 \%$ of the chips and $15 \%$ of its chips are defective. A chip is selected at random and found to be defective. What is the probability that it came from Machine I?
(c) If a publisher of nontechnical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is .005 and errors are independent from page to page, what is the probability that one of its 400-page novels will contain (i) exactly one page with errors? (ii)At most three pages with errors?
Q. 5 (a) Samples of sizes 10 and 14 were taken from two normal populations with standard deviation 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. Test whether the means of the two populations are the same at $5 \%$ level. [ $t_{0.05}=2.0739$ ].
(b) Two independent samples of 8 and 7 items respectively had the following values of the variable (weight in kg ):

| Sample I : | 9 | 11 | 13 | 11 | 15 | 9 | 12 | 14 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sample II: | 10 | 12 | 10 | 14 | 9 | 8 | 10 |  |

Do the two estimates of population variance differ significantly? Given that for $(7,6)$ d.f. the value of F at $5 \%$ level of significance is 4.20 nearly.
(c) Records taken of the number of male and female births in 830 families having four children are as follows:

| Number of male births | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of female births | 4 | 3 | 2 | 1 | 0 |
| Number of families | 32 | 178 | 290 | 236 | 94 |

Test whether the data are consistent with the hypothesis that the Binomial law holds and the chance of male birth is equal to that of female birth, namely $\mathrm{p}=\mathrm{q}=1 / 2$. [ $\chi^{2}$ at $5 \%$ level of significance for 4 df is 9.49]

OR

Q. 5 (a) Two samples of size 9 and 8 give the sum of squares of deviations from their respective means equal 160 inches and 91 inches square respectively. Can they be regarded as drawn from two normal populations with the same variance?
( F for 8 and 7 d.f. $=3.73$ ).
(b) A die is thrown 276 times and the results of these throws are given below:

| Number appeared on the die | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 40 | 32 | 29 | 59 | 57 | 59 |

Test whether the die is biased or not. [ $X^{2}$ at $5 \%$ level of significance for 5 df is 11.09]
(c) The following figures refer to observations in live independent samples:

| Sample I: | 25 | 30 | 28 | 34 | 24 | 20 | 13 | 32 | 22 | 38 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sample II: | 40 | 34 | 22 | 20 | 31 | 40 | 30 | 23 | 36 | 17 |

Analyse whether the samples have been drawn from the population of equal means.
[ t at $5 \%$ level of significance for 18 d.f is 2.1 ] Test whether the means of two population are same at $5 \%$ level ( t at $0.05=2.0739$ )

