## Subject Name: Dynamics of Linear Systems

 Time: 02:30 PM TO 05:00 PM1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
Q. 1 (a) (i) Define system. 03
(ii) List out the types of system.
(b) Explain convolution property of z-transform.

04
(c) Consider the RC circuit given in the figure below. Assume 07 that the circuit's time constant is $\mathrm{RC}=1 \mathrm{~s}$. The impulse response of this circuit is given by $h(t)=e^{-t} u(t)$.

Determine the voltage across the capacitor, $\mathrm{y}(\mathrm{t})$, resulting from an input voltage $x(t)=u(t)-u(t-2)$.

Q. 2 (a) Use the convolution property to find the FT of the system output $\mathrm{Y}(\mathrm{j} \omega)$ for the following inputs and system impulse response:

$$
x(t)=3 e^{-t} u(t) \text { and } h(t)=2 e^{-2 t} u(t)
$$

(b) Use the convolution property to find the time-domain signal corresponding to the following frequency-domain representation:

$$
X\left(e^{j \Omega}\right)=\left(\frac{1}{1-\left(\frac{1}{2}\right) e^{-j \Omega}}\right)\left(\frac{1}{1+\left(\frac{1}{2}\right) e^{-j \Omega}}\right)
$$

(c) Evaluate the periodic convolution of the sinusoidal signal

$$
z(t)=2 \cos (2 \pi t)+\sin (4 \pi t)
$$

with the periodic square wave $\mathrm{x}(\mathrm{t})$ as shown below:

(c) The output of an LTI system in response to an input $x(t)=$
$e^{-2 t} u(t)$ is $y(t)=e^{-t} u(t)$. Find the frequency response and the impulse response of this system.
Q. 3 (a) Find the DTFT of $\mathrm{x}[\mathrm{n}]=\delta[\mathrm{n}]$

$$
x(t)=\sum_{t=-\infty}^{\infty} \delta(\mathrm{t}-4 \mathrm{l})
$$

(c) Prove the following properties in context of Continuous

Time Fourier Transform:
(i) Time shifting
(ii) Time and frequency scaling

## OR

Q. 3 (a) State Dirichlet condition for Fourier series representation.03
(b) Prove the duality property of Fourier transform. $\mathbf{0 4}$
(c) Determine the appropriate Fourier representations of the 07 following time domain signals:
(i) $\mathrm{x}(\mathrm{t})=\mathrm{e}^{-\mathrm{t}} \cos (2 \pi \mathrm{t}) \mathrm{u}(\mathrm{t})$
(ii) $x(t)=|\sin (2 \pi t)|$
Q. 4 (a) Explain the linearity property of Laplace transform. 03
(b) Derive the relationship between Laplace transform and 04 Fourier transform.
(c) Analyze the role of Region of Convergence (ROC) for $\mathbf{0 7}$ defining the stability of system in the context of Laplace transform.

## OR

Q. 4 (a) Explain the modulation property in context of Fourier transform.
(b) Explain the differencing and summation property of $\mathbf{0 4}$ discrete Fourier transform.
(c) Find the inverse Discrete Time Fourier Transform (DTFT) of

$$
X\left(e^{j \Omega}\right)=\frac{-\frac{5}{6} e^{-j \Omega}+5}{1+\frac{1}{6} e^{-j \Omega}-\frac{1}{6} e^{-j \Omega 2}}
$$

Q. 5 (a) Explain the linearity property of z-transform.
(b) Explain the concept of poles and zeros with respect to z- 04 transform.
(c) Determine the z-transform of the signal

$$
x[n] \xlongequal{\leftrightharpoons}-u[-n-1]+\left(\frac{1}{2}\right)^{n} u[n]
$$

Depict the ROC and the locations of poles and zeros of $\mathrm{X}(\mathrm{z})$ in the z-plane.

## OR

Q. 5 (a) Explain the initial value theorem in conext of z-transform. 03
(b) Determine the z-transform of the signal 04

$$
x[n]=\alpha^{n} u[n]
$$

(c) Find the inverse $z$-transform of

$$
\begin{aligned}
& X(z)=\frac{2+z^{-1}}{1-\frac{1}{2} z^{-1}} \\
& \text { with ROC }|z|>\frac{1}{2}
\end{aligned}
$$

