Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (Old) EXAMINATION – WINTER 2019

Subject Code: 130001 **Date: 22/11/2019**

Subject Name: Mathematics-III

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1

(a) Obtain Fourier series to represent $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in the interval $0 < x < 2\pi$. **07**

(b) Using Laplace Transform solve the given IVP. **07**
$y''-y=t, y(0)=1, y'(0)=1$.

Q.2 **(a)** Solve by Method of Variation of Parameters. $y''+4y=\sec 2x$. **07**

(b) Solve $(3x+2)^2 y''+3(3x+2)y'-36y=3x^2+4x+1$. **07**

OR

(b) Obtain series solution of $y''-xy=0$. **07**

Q.3 **(a)**

(1) Solve linear differential equation $\frac{dy}{dx}+2xy=2e^{-x^2}$. **03**

(2) Solve $\frac{dy}{dx}+\frac{y}{x}\log x=\frac{y}{x^2}(\log y)^2$. **04**

(b) Using method of undermined co-efficient method to solve $y''+2y'+10y=25x^2+3$. **07**

OR

Q.3 **(a)** (1) Solve $(x^2y^2+xy+1)ydx+(x^2y^2-xy+1)xdy=0$. **03**

(2) Solve $(1+y^2)dx+(x-e^{\tan^{-1}y})dy=0$. **04**

(b) Using method of undermined co-efficient method to solve $y''-9y=x+e^{2x}-\sin 2x$. **07**

Q.4

(a) (1) Find the inverse Laplace Transform of $\frac{5s^2+3s-16}{(s-1)(s-2)(s-3)}$. **03**

(2)Using convolution theorem, find the inverse Laplace transform of $\frac{s+2}{(s^2+4s+5)^2}$. **04**

(b) (1) Find the Fourier Transform of the function $f(x) = e^{-ax^2}$. **04**

(2) Evaluate $\int_0^\infty x^n e^{-\sqrt{ax}} dx$. **03**

Q.4 **(a)** (1) By using first shifting theorem, obtain the value of $L\{(t+1)^2 e^t\}$. **03**

(2) Find $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$. **04**

(b) (1)Find the Fourier Cosine transform of the function $f(x) = \begin{cases} k & 0 < x < a \\ 0 & x > a \end{cases}$. **04**

(2) Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$. **03**

Q.5 **(a)** Prove that (1) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ (2) $J_{\frac{-1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ **07**

(b) A taut string of length l has its ends $x = 0$ and $x = l$ fixed. The mid-point is stretched to a small height and released from rest at time $t = 0$. Find the displacement $u(x,t)$. **07**

OR

Q.5 **(a)** Show that $\int_{-1}^{1} P_m(x) P_n(x) dx = 0$, if $m \neq n$ and $\int_{-1}^{1} P_n^2(x) dx = \frac{2}{2n+1} (n = 0,1....)$ if $m = n$. (m, n being integers) **07**

(b) Solve $u_{xx} + u_{yy} = 0$ which satisfies the boundary condition $u(0,y) = u(a,y) = o$ for $0 \le y \le b$ and $u(x,b) = 0, u(x,0) = f(x)$ for $0 \le x \le a$. **07**

****