

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER- IV (New) EXAMINATION – WINTER 2019

Subject Code: 2140001
Date: 07/12/2019
Subject Name: Mathematics-4
Time: 10:30 AM TO 01:30 PM
Total Marks: 70
Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q-1** (a) Find the principal argument of $z = \frac{1-7i}{(2+i)^2}$ 3
- (b) If $f(z) = \frac{x^3-y^3}{x^3+y^3}$ if $(x, y) \neq (0,0)$ 4
 $= 0$ if $(x, y) = (0,0)$
 Show that $f(z)$ is not continuous at the origin.

- (c) Solve the following system of linear equations by Gauss-elimination method. $x + y + z = 9, 2x - 3y + 4z = 13, 3x + 4y + 5z = 40$. 7

- Q-2** (a) Check whether the function $f(z) = \bar{z}$ is analytic or not? 3
- (b) Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$. 4
- (c) Determine the mobius transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively. 7

OR

- (c) Find real and imaginary parts of $(-1 - i)^7 + (-1 + i)^7$ 7

- Q-3** (a) Prove that $\oint_C \frac{\sin 3z}{z + \frac{\pi}{2}} dz = 2\pi i$, where C is the circle $|z| = 5$. 3
- (b) Expand $f(z) = \frac{1-e^z}{z}$ in Laurent's series about $z = 0$ and identify singularity. 4

- (c) Use residues to evaluate $\int_0^\infty \frac{x^2 dx}{(x^2+1)(x^2+4)}$ 7

OR

- (a) Find the radius of convergence of $\sum_{n=1}^\infty \left(1 + \frac{1}{n^2}\right)^{n^3} z^n$. 3

- (b) Evaluate $\oint_C (x^2 - iy^2) dz$ along the parabola $y = 2x^2$ from (1,2) to (2,8). 4

- (c) Expand $f(z) = \frac{1}{(z+2)(z+4)}$ valid for the regions (i) $|z| < 2$, 7
 (ii) $2 < |z| < 4$, (iii) $|z| > 4$.

- Q-4** (a) Prove that $\Delta \ln f(x) = \ln \left[1 + \frac{\Delta f(x)}{f(x)} \right]$ 3

- (b) Find a real root of the equation $x^3 + 4x^2 - 1 = 0$ by using bisection method correct up to two decimal places. 4
- (c) Determine the interpolating polynomial of degree three using Lagrange's interpolation formula for the table below. 7

x	-1	0	1	3
y	2	1	0	-1

OR

- (a) Use trapezoidal rule to estimate $\int_{0.5}^{1.3} e^{x^2} dx$ using a strip of width 0.2. 3
- (b) Evaluate $I = \int_0^1 \frac{dt}{1+t}$ by one point, Gaussian formula. 4
- (c) Solve the following equations by Gauss-Seidel method correct up to two decimal places $20x + 2y + z = 30, x - 40y - 3z = -75, 2x - y + 10z = 30$. 7

- Q5** (a) Compute $\cosh(0.56)$ using Newton's forward difference formula for the following table. 3

X	0.5	0.6	0.7	0.8
F(X)	1.127626	1.185465	1.255169	1.337435

- (b) Using Newton's divided difference interpolation formula compute $f(9.2)$ from the following data. 4

x	8.0	9.0	9.5	11.0
f(x)	2.079442	2.197225	2.251292	2.397895

- (c) Using improved Euler's method, solve $y' = 1 - y$ with the initial condition $y(0) = 0$ and tabulate the solutions at $x = 0.1, 0.2$. compare the answer with the exact solution. 7

OR

- (a) Using N-R method find an iterative formula to find $\sqrt{N}$ (where N is positive number) and hence find $\sqrt{5}$. 3
- (b) Evaluate the integral $\int_4^{5.2} \log_e x dx$, using Simpson's $\frac{3}{8}$ th rule. 4
- (c) Find the largest eigen value and the corresponding eigenvector for $A =$ 7

$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$