Seat No.: ________ Enrolment No.

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (NEW) EXAMINATION – WINTER 2019

Subject Code: 3110015 **Date: 01/01/2020**

Subject Name: Mathematics –2

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Marks

Q.1 **(a)** Find the length of curve of the portion of the circular helix $\vec{r}(t)=\cos t\,\hat{i}+\sin t\,\hat{j}+t\,\hat{k}$ from $t=0$ to $t=\pi$ **03**

(b) $\int_{(1,2)}^{(3,4)}\left(xy^2+y^3\right)dx+\left(x^2y+3xy^2\right)dy$ is independent of path joining the points (1, 2) and (3,4). Hence, evaluate the integral. **04**

(c) Verify tangential form of Green's theorem for $\vec{F}=\left(x-\sin y\right)\hat{i}+\left(\cos y\right)\hat{j}$, where C is the boundary of the region bounded by the lines $y=0, x=\pi/2$ and $y=x$. **07**

Q.2 **(a)** Find the Laplace transform of $f(t)$ defined as **03**

$$f(t)=\frac{t}{k} \qquad 0<t<k$$
$$=1 \qquad t>k$$

(b) Find the inverse Laplace transform of $\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$ **04**

(c) (i) Calculate the curl of the vector $xyz\,\hat{i}+3x^2y\,\hat{j}+\left(xz^2-y^2z\right)\hat{k}$ **07**

(ii) The temperature at any point in space is given by $T=xy+yz+zx$. Determine the derivative of T in the direction of the vector $3\hat{i}-4\hat{k}$ at the point (1, 1, 1).

OR

(c) Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, $r=|\vec{r}|$, and $\bar{a}$ is a constant vector. Find the value of $div\left(\frac{\bar{a}\times\bar{r}}{r^n}\right)$ **07**

Q.3 **(a)** Find constants a, b and c such that $\vec{V}=\left(x+2y+az\right)\hat{i}+\left(bx-3y-z\right)\hat{j}+\left(4x+cy+2z\right)\hat{k}$ is irrotational. **03**

(b) Using Fourier cosine integral representation show that **04**

$$\int_0^\infty \frac{\cos\omega x}{k^2+\omega^2}d\omega=\frac{\pi e^{-kx}}{2k}$$

(c) Solve the following differential equations: **07**

(i) $\cos(x+y)\,dy=dx$

(ii) $\sec^2 y\frac{dy}{dx}+x\tan y=x^3$

OR

Q.3 **(a)** Find the Laplace transform of (i) $\int_0^t \frac{\sin t}{t} dt$ (ii) $t^2 u(t-3)$ **03**

(b) Using Convolution theorem obtain $L^{-1}\left(\frac{1}{s(s^2+a^2)}\right)$ **04**

(c) Find the power series solution of $\frac{d^2y}{dx^2} + xy = 0$ **07**

Q.4 **(a)** Find the Laplace transform of the waveform **03**

$f(t) = \left(\frac{2t}{3}\right), 0 \le t \le 3$

(b) Using the Laplace transforms, find the solution of the initial value problem $y'' + 25y = 10\cos 5t \quad y(0) = 2, y'(0) = 0$ **04**

(c) Using variation of parameter method solve $(D^2+1)y = x\sin x$ **07**

OR

Q.4 **(a)** Solve $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx}$ **03**

(b) Solve $y''' - 3y'' + 3y' - y = 4e^t$ **04**

(c) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$ using method of undetermined coefficients. **07**

Q.5 **(a)** Classify the singular points of the equation $x^3(x-2)y'' + x^3y' + 6y = 0$ **03**

(b) Solve $(D^2+4)y = \cos 2x$ **04**

(c) Solve (i) $ye^x dx + (2y + e^x)dy = 0$ (ii) $\frac{dy}{dx} + 2y\tan x = \sin x$ **07**

OR

Q.5 **(a)** Solve $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$ **03**

(b) If $y_1 = x$ is one of solution of $x^2y'' + xy' - y = 0$ find the second solution. **04**

(c) Using Frobenius method solve $x^2y'' + 4xy' + (x^2+2)y = 0$ **07**

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