Seat No.: _____ Enrolment No.

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (SPFU) EXAMINATION – WINTER 2019

Subject Code: MTH001 **Date: 06/01/2020**

Subject Name: Calculus

Time: 10:30 AM TO 01:00 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Test the convergence of $\sum_{n=1}^{\infty} \frac{5n^3-3n}{n^2(n-2)(n^2+5)}$. **07**

(b) Test the convergence of $\frac{1}{1\bullet 2\bullet 3}+\frac{x}{4\bullet 5\bullet 6}+\frac{x^2}{7\bullet 8\bullet 9}+.....$ **07**

Q.2 **(a)** Trace the curve $r=a(1+\cos\theta), a>0$. **07**

(b) Sketch the region of integration and evaluate $\int_1^4\int_0^{\sqrt{x}} \frac{3}{2}e^{y/\sqrt{x}}dydx$ **07**

Q.3 **(a)** 1. If $(\cos x)^y=(\sin y)^x$, find $\frac{dy}{dx}$. **07**

2. If $x^5+y^5=5x^2$, find $\frac{dy}{dx}$.

(b) Evaluate $\iint_R r\sqrt{a^2-r^2}drd\theta$ over the upper half of the circle $r=a\cos\theta$. **07**

Q.4 **(a)** Find the minimum and maximum value of $f(x,y)=x^3+y^3-3xy$. **07**

(b) Evaluate $\int_0^2\int_{y/2}^1 e^{x^2}dxdy$ by changing order of integration. **07**

Q.5 **(a)** If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}=2\cos 3u\sin u$. **07**

(b) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyzdzdydx$. **07**

Q.6 **(a)** Find the equation of tangent plane and normal line at the point (-2,1,-3) to the ellipsoid $\frac{x^2}{4}+y^2+\frac{z^2}{9}=3$. **07**

(b) If $u=f(r)$, where $r^2=x^2+y^2$, prove that $u_{xx}+u_{yy}=f''(r)+\frac{1}{r}f'(r)$. **07**

Q.7 **(a)** Test the convergence for $\sum_{n=1}^{\infty}\frac{2\tan^{-1}n}{n^2+1}$. **07**

(b) Find the area of the region R enclosed by the parabola $y=x^2$ and the line $y=x+2$. **07**

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