Seat No.: _____  Enrolment No. 

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER– IV (New) EXAMINATION – WINTER 2019**

**Subject Code: 2141905** **Date: 07/12/2019**

**Subject Name: Complex Variables and Numerical Methods**

**Time: 10:30 AM TO 01:30 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q-1** **(a)** Determine whether the function $\begin{cases} \frac{z^2+3iz-2}{z+i} ; & z \neq -i \\ 5 ; & z = -i \end{cases}$ is continuous? **03**

Can the function be redefined to make it continuous at $z = -i$?

**(b)** State De Moivre's Theorem. Find the roots of the equation $z^4 + 1 = 0$. **04**

**(c)** Solve the following system of equations using Gauss Seidel Method correct to four decimal places. **07**

$$30x - 2y + 3z = 75;\ \ 2x + 2y + 18z = 30; x + 17y - 2z = 48$$

**Q-2** **(a)** Check whether the function $f(z) = e^{z^2}$ is entire or not. Also find derivative of $f(z)$. **03**

**(b)** Find the bilinear transformation which maps $z = 1, 0, -1$ into the points $w = i, \infty, 1$. **04**

**(c)** Using the Residue Theorem Evaluate, $\int_0^{2\pi} \frac{d\theta}{5-3sin\theta}$ **07**

**OR**

**(c)** Show that the function $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic. Find the conjugate harmonic function $v$ and express $u + iv$ as analytic function of $z$ **07**

**Q-3** **(a)** Evaluate $\oint_c \frac{z^2+1}{z^2-1}\, dz$ if $c$ is the circle of unit radius with centre at $z = 1$. **03**

**(b)** Find the real part and imaginary part of $\sqrt{i}^{\sqrt{i}}$ **04**

**(c)** Evaluate $\int f(z)dz$ where $f(z)$ is defined by **07**

$$f(z) = \begin{cases} 1 \ \ when\ y < 0 \\ 4y \ \ when\ y > 0 \end{cases}$$

And C is the arc from $z = -1 - i$ to $z = 1 + i$ along the curve $y = x^3$.

**OR**

**Q-3** **(a)** Find the type of singularity of the function $f(z) = \frac{e^{2z}}{(z-1)^4}$ **03**

(b) Find and Sketch the region of the z-plane for which $1 < x < 2$ under the transformation $w = \frac{1}{z}$ 04

(c) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ valid for region 07
(i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

**Q-4** (a) Use Euler's Method, find $y(0.2)$ given that $\frac{dy}{dx} = x - y^2$ ; $y(0) = 1$ take $h = 0.1$ 03

(b) Evaluate $\sqrt{8}$ to two decimal places by Newton's iterative formula. 04

(c) Determine the polynomial by Newton's forward difference formula from the following table 07

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$ | -10 | -8 | -8 | -4 | 10 | 40 |

**OR**

**Q-4** (a) Solve the following system ofequation using Gauss Elimination Method 03
$x + y + z = 7;\ 3x + 3y + 4z = 24;\ 2x + y + 3z = 16$

(b) Use Secant Method to find the root of $f(x) = x log_{10} x - 1.9 = 0$ 04

(c) Using Newton's Divided Differences formula to find a polynomial function, satisfying the following data. 07

| $x$ | -4 | -1 | 0 | 2 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 1245 | 33 | 5 | 9 | 1335 |

**Q-5** (a) Evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ by using Gaussian formula for $n = 2$ and $n = 3$ 03

(b) Use fourth order Range-Kutta method to compute $y(0.2)$ and $y(0.4)$ given that $\frac{dy}{dx} = y - \frac{2x}{y}$; $y(0) = 1$. 04

(c) Find the dominant Eigen value of $A = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$ by Power Method and the corresponding Eigen vector. 07

**OR**

**Q-5** (a) State Trapezoidal Rule and evaluate $\int_0^1 e^x\, dx$ using it with $n = 10$ 03

(b) Use Lagrange's formula to fit a polynomial to the data 04

| $x$ | -1 | 0 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 8 | 3 | 1 | 12 |

(c) Apply improved Euler's method to solve the initial value problem 07
$y' = x + y$ with $y(0) = 0$ choosing $h = 0.2$ and compute $y_1, y_2, \ldots, y_5$.

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