Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (OLD) EXAMINATION – WINTER 2019

Subject Code: 110008 **Date: 17/01/2020**

Subject Name: Maths - I

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q-1	**(a)**	(i) State Sandwich theorem, using it find $\lim_{x\to 0} g(x)$ if $3-x^2 \le g(x) \le 3\sec x$ for all x.	**4**
		(ii) Can Rolle's theorem for $f(x)=\|x\|, x\in[-1,1]$ applied?	**3**
	(b)	If $u = f(x-y, y-z, z-x)$, Prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$	**7**
Q-2	**(a)**	(i) Use Taylor's series to find the expansion of $\log_e x$ in powers of $(x-1)$.	**4**
		(ii) Use L'Hospital rule, Evaluate $\lim_{x\to 0}\frac{\log x}{\cot x}$	**3**
	(b)	Trace the curve $r^2 = a^2\cos 2\theta$.	**7**
Q-3	**(a)**	(i) Test the convergence of $\sum_{n=0}^{\infty}\frac{2^n-1}{3^n}$.	**4**
		(ii) Does the sequence $\left\{\frac{3}{n+3}\right\}$ monotone?	**3**
	(b)	If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ Prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\cos 3u\sin u.$	**7**
Q-4	**(a)**	(i) Test the convergence of $\sum_{n=1}^{\infty}\frac{3^n n!}{n^n}$, by Ratio Test.	**4**
		(ii) Discuss the convergence if the series $4-9+5+4-9+5+4-9+5.....\infty$.	**3**
	(b)	Find the extremum values for $f(x,y)=x^3+y^3-3xy$.	**7**
Q-5	**(a)**	(i) Expand $x^2y+3y-2$ in the neighbourhood of the point $(1,-2)$.	**4**
		(ii) Find the equation for tangent plane and normal line at the point $(1,1,1)$ on the surface $x^2+y^2+z^2=3$.	**3**
	(b)	Find the Volume of sphere $x^2+y^2+z^2=a^2$.	**7**
Q-6	**(a)**	(i) Evaluate $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}dA$, by changing the order of integration.	**4**
		(ii) Find the value of m if $\bar{F}=(x+2y)i+(my+4z)j+(5z+6x)k$ is solenoidal.	**3**
	(b)	Evaluate $\iint_R (x+y)dydx$, where R is the region bounded by $x=0, x=2, y=x, y=x+2$.	**7**
Q-7	**(a)**	(i) Evaluate $\int_0^1\int_0^{\sqrt{z}}\int_0^{2\pi}(r^2\cos^2\theta+z^2)rd\theta drdz$	**4**

(ii) Using Green's theorem to evaluate the integral $\oint_C \left(y^2dx + x^2dy\right)$, where C: The triangle bounded by $x=0, x+y=1, y=0$. **3**

(b) Use divergence theorem to evaluate $\iint_S \left(x^3dydz + x^2ydzdx + x^2zdxdz\right)$ where S is the closed surface consisting of the cylinder $x^2+y^2=a^2$ and the circular discs $z=0$ and $z=b$. **7**

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