Seat No.: ________ Enrolment No. ________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (OLD) EXAMINATION – WINTER 2019

Subject Code: 110014 **Date: 17/01/2020**

Subject Name: Calculus

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Test the convergence of the sequence $\left(\dfrac{n^2+n}{2n^2-n}\right)$. **03**

(ii) Expand $\log x$ in powers of $(x-1)$. **04**

(b) (i) Evaluate $\lim\limits_{x\to 0}(\cos x)^{\cot x}$ **03**

(ii) Test the convergence of the series $\sum\limits_{n=1}^{\infty}\sin\dfrac{1}{n}$ **04**

Q.2 **(a)** (i) Evaluate $\lim\limits_{x\to 2}\left[\dfrac{1}{x-2}-\dfrac{1}{\log(x-1)}\right]$ **03**

(ii) Determine the interval of convergence for the series $\sum\limits_{n=1}^{\infty}\dfrac{x^n}{2^n}, x>0$ **04**

(b) Expand $\log\cos\left(x+\dfrac{\pi}{4}\right)$ using Taylor`s theorem in ascending powers of x and hence find the value of $\log(\cos 48^0)$ correct up to three decimal places. **07**

Q.3 **(a)** (i) Evaluate $\int_0^1 \dfrac{x^9}{\sqrt{1-x^2}}dx$ **03**

(ii) Test the convergence of the improper integral $\int_4^{\infty}\dfrac{3x+5}{x^4+7}dx$ **04**

(b) (i) Show that $f(x,y)=\dfrac{xy}{\sqrt{x^2+y^2}}, \quad (x,y)\neq(0,0)$
$\qquad\qquad = 0, \quad (x,y) = (0,0)$ is continuous at origin. **04**

(ii) If $u=e^{xyz}$, show that $\dfrac{\partial^3 u}{\partial x\partial y\partial z}=\left(1+3xyz+x^2y^2z^2\right)e^{xyz}$ **03**

Q.4 **(a)** If $u=f(r)$ and $r^2=x^2+y^2+z^2$, prove that **07**

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f''(r)+\frac{2}{r}f'(r)$$

(b) (i) If $u=f\left(\dfrac{x}{y},\dfrac{y}{z},\dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z}=0$ **04**

(ii) Find the equations of the tangent plane and normal line to the surface $z=2x^2+y^2$ at the point $(1,1,3)$ **03**

Q.5 **(a)** If $u = \cos^{-1}\left(\dfrac{x^3+y^3}{x+y}\right)$,show that **07**

(i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = -2\cot u$

(ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \dfrac{\sin 2u \sin u - 4\cos u}{\sin^3 u}$

(b) (i) Trace the curve $x^3 + y^3 = 3axy, a > 0$ **04**

(ii) Discuss the maxima and minima of the function $x^2 + y^2 + 6x + 12$ **03**

Q.6 **(a)** (i) Evaluate $\displaystyle\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dxdy}{1+x^2+y^2}$ **03**

(ii) Evaluate $\displaystyle\int_0^\infty \int_x^\infty e^{-y^2}\, dy\, dx$,by changing the order of integration. **04**

(b) The temperature at any point (x, y, z) in space is $T = 400xyz^2$ .Find the highest temperature on the surface of unit sphere $x^2 + y^2 + z^2 = 1$ by the method of Lagrange`s multipliers. **07**

Q.7 **(a)** Evaluate $\displaystyle\iint_R (x^2 + y^2)dA$, by changing the variables, where R is the region lying in the first quadrant and bounded by the hyperbolas $x^2 - y^2 = 1, x^2 - y^2 = 9, xy = 2$ and $xy = 4$ **07**

(b) Evaluate $\displaystyle\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} e^{-(x^2+y^2+z^2)^{3/2}} dxdydz$ **07**

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