Seat No.: ________ Enrolment No.

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (OLD) EXAMINATION – WINTER 2019

Subject Code: 110015 **Date: 01/01/2020**

Subject Name: Vector Calculus And Linear Algebra

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (1) Find the Rank of $\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ by row echelon form. **03**

(2) Solve the following system of equation by Gauss elimination method. **04**

$x+y+2z=9$

$2x+4y-3z=1$.

$3x+6y-5z=0$

(b) Determine whether the set V of all pairs of real numbers (x,y) with the operations $(x_1,y_1)+(x_2,y_2)=(x_1+x_2+1, y_1+y_2+1)$ and $k(x,y)=(kx,ky)$ is a vector space. **07**

Q.2 (a) (1) Find the inverse of $A=\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ using Gauss-Jordan method, if exists. **03**

(2) Determine whether $V=R^3$ is an inner product space under the inner product $\langle u,v\rangle = 2u_1v_1+u_2v_2+4u_3v_3$. **04**

(b) Evaluate $\iint_S \bar{F}\cdot\hat{n}dS$ using Gauss divergence theorem where $\bar{F}=4xz\hat{i}+xyz^2\hat{j}+3z\hat{k}$ over the region bounded by the cone $z^2=x^2+y^2$ and plane $z=4$, above the xy plane. **07**

Q.3 (a) (1) Find the directional derivative of $\phi = xy^2+yz^3$ at $(2,-1,1)$ in the direction of the normal to the surface $x\log z - y^2 = -4$ at $(-1,2,1)$. **03**

(2) Show that $\bar{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ is conservative. Find its scalar potential ϕ. **04**

(b) Let $T: M_{22}\to R$ be a linear transformation for which $T(v_1)=1$, $T(v_2)=2$, $T(v_3)=3$, $T(v_4)=4$ where $v_1=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, v_2=\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, v_3=\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, v_4=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, Find $T\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $T\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. **07**

Q.4 (a) (1) If $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$, show that $div(r^n\bar{r})=(n+3)r^n$. **03**

(2) Show that $\bar{F} = \left(y^2 - z^2 + 3yz - 2x\right)\hat{i} + \left(3xz + 2xy\right)\hat{j} + \left(3xy - 2xz + 2z\right)\hat{k}$ is both solenoidal and irrotational. **04**

(b) Let R^3 have the Euclidean inner product. Use Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis. Where $u_1 = (1,0,0)$, $u_2 = (3,7,-2)$ and $u_3 = (0,4,1)$. **07**

Q.5 **(a)** (1) Find a basis for the subspace of P_2 spanned by the vectors $1+x, x^2$, $-2+2x^2$, $-3x$. **03**

(2) Determine whether the linear transformation $T: R^2 \to P_1$, $T(a,b) = a + (a+b)x$ is one-to-one and onto. **04**

(b) Verify Stokes' theorem for $\bar{F} = (x+y)\hat{i} + (y+z)\hat{j} - x\hat{k}$ and S is the surface of the plane $2x + y + z = 2$ which is in the first octant. **07**

Q.6 **(a)** (1) Find the least-square solution of the linear system $Ax = b$ given by **03**

$$\begin{aligned} x_1 + x_2 &= 7 \\ -x_1 + x_2 &= 0 \\ -x_1 + 2x_2 &= -7 \end{aligned}.$$

(2) Determine whether b is in the column space of A, and if so, express b as a linear combination of the column vectors of A if $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, $b = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$. **04**

(b) Verify Cayley-Hemilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}. Also express $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$ as a linear polynomial in A. **07**

Q.7 **(a)** (1) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ along the parabola $y^2 = x$ between the point $(0,0)$ and $(1,1)$ where $\bar{F} = x^2\hat{i} + xy\hat{j}$. **03**

(2) (i) If $f(x,y,z) = 3x^2y - y^3z^2$, find grad f at the point $(1,-2,-1)$. **04**

(ii) Find unit normal vector to the surface $x^2y + 2xz^2 = 8$ at the point $(1,0,2)$.

(b) Find a matrix P that diagonalizes $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. **07**

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