

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER- VI (New) EXAMINATION – WINTER 2019

Subject Code: 2160704

Date: 09/12/2019

Subject Name: Theory of Computation

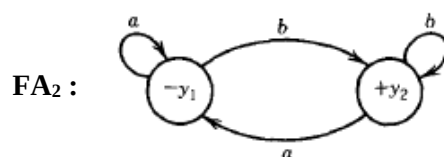
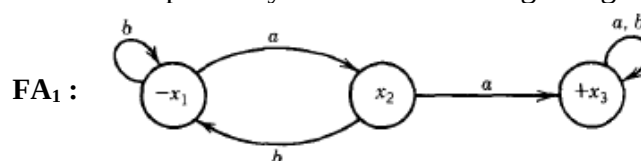
Time: 02:30 PM TO 05:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

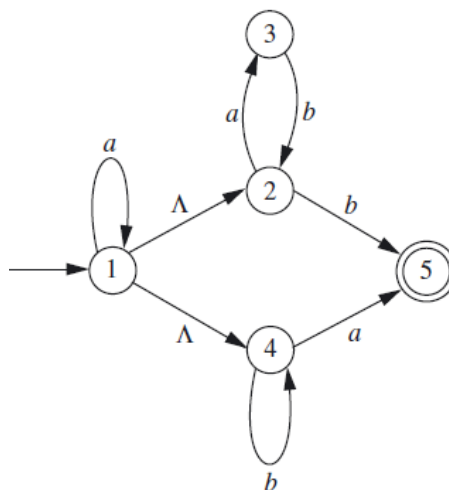
- | | MARKS |
|---|-----------|
| Q.1 (a) Define – bijection function. Check whether the function $f : Z \rightarrow Z$ defined by $f(x) = 2x$ is a bijection function or not. Justify your answer. | 03 |
| (b) Draw an FA that recognizes the language of all strings containing even no of 0's and even no of 1's over $\Sigma = \{0,1\}$. Also write a regular expression for the same language. | 04 |
| (c) Write the principle of Mathematical Induction. Prove using mathematical induction that for every $n \geq 0$, | 07 |
| $\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$ | |
| (Consider the sum on the left is 0 for $n = 0$) | |
| Q.2 (a) Find regular expression and also derive the words corresponding to the language defined recursively below over $\Sigma = \{a, b\}$. | 03 |
| i. $a \in L$
ii. For any $x \in L$, xa and xb are elements of L | |
| (b) Define – Equivalence relation. A relation on the set $\{1,2,3\}$ is given as $R = \{(a, b) \mid a - b \text{ is an even no}\}$. Check whether R is equivalence relation or not. Give reasons. | 04 |
| (c) Give transition table for PDA recognizing the following language and trace the move of the machine for input string $abcba$:
$L = \{xcx^r \mid x \in \{a, b\}^*\}$ | 07 |
| OR | |
| (c) Give transition table for PDA accepting the language of all odd-length strings over $\{a, b\}$ with middle symbol a . Also draw a PDA for the same. | 07 |
| Q.3 (a) Let FA_1 and FA_2 be the FAs as shown in the figure recognizing the languages L_1 and L_2 respectively. Draw an FA recognizing the language, $L_1 \cup L_2$. | 03 |



- (b) Define – Moore machine. Convert the following Moore machine into its equivalent Mealy machine:

Old state	After input a	After input b	Output
	New state	New state	
$-q_0$	q_1	q_2	0
q_1	q_3	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_3	1

- (c) Convert the following NFA - Λ into its equivalent DFA that accepts the same language: 07



OR

- Q.3** (a) Prove that – “If there is a CFG for the language L that has no Λ -productions, then there is a CFG for L with no Λ -productions and no unit productions”. Support your answer with the help of the following CFG: 03

$S \rightarrow A \mid bb$
 $A \rightarrow B \mid b$
 $B \rightarrow S \mid a$

- (b) Write CFG for the following languages : 04

- i. $\{a^i b^j c^k \mid i = j + k\}$
 ii. $\{a^i b^j c^k \mid j = i \text{ or } j = k\}$

- (c) Define – ambiguous grammar, leftmost derivation. Check whether the following grammars are ambiguous or not. Justify your answer with proper reason. 07

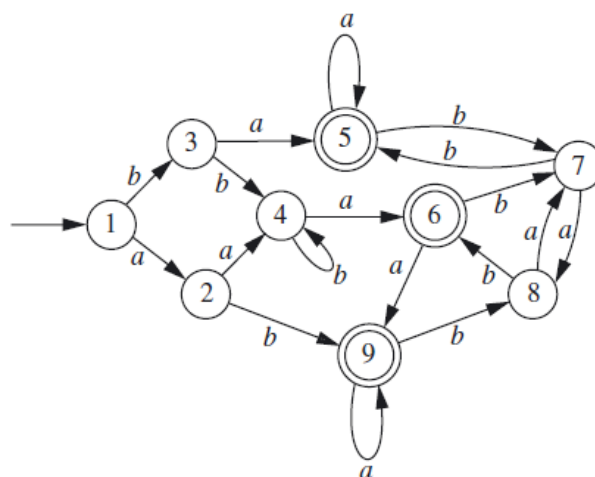
- i. $S \rightarrow ABA$
 $A \rightarrow aA \mid \Lambda$
 $B \rightarrow bB \mid \Lambda$
- ii. $S \rightarrow A \mid B$
 $A \rightarrow aAb \mid aabb$
 $B \rightarrow abB \mid \Lambda$

- Q.4** (a) Describe the language generated by the following grammars: 03

- i. $S \rightarrow aA \mid bC \mid b$
 $A \rightarrow aS \mid bB$
 $B \rightarrow aC \mid bA \mid a$
 $C \rightarrow aB \mid bS$
- ii. $S \rightarrow aT \mid bT \mid \Lambda$
 $T \rightarrow aS \mid bS$

- (b) Discuss – Nondeterministic Turing Machines and Universal Turing Machines 04

- (c) Find a minimum-state FA for the following FA that recognizes the same language using the minimization algorithm: 07



OR

- Q.4** (a) Find the CFG for the regular expression : $(011 + 1)^* (01)^*$ 03
 (b) Prove that the language $L = \{a^n b^n a b^{n+1} \mid n = 1, 2, 3, \dots\}$ is nonregular using pumping lemma. 04
 (c) Convert the following CFG into its equivalent CNF: 07

$S \rightarrow TU \mid V$
 $T \rightarrow aTb \mid \Lambda$
 $U \rightarrow cU \mid \Lambda$
 $V \rightarrow aVc \mid W$
 $W \rightarrow bW \mid \Lambda$

- Q.5** (a) Convert the following CFG into its equivalent PDA. 03

$S \rightarrow AB$
 $A \rightarrow BB$
 $B \rightarrow AB$
 $A \rightarrow a$
 $B \rightarrow a \mid b$

- (b) Show using the pumping lemma that the following language is not a CFL. 04
 $L = \{a^i b^j c^k \mid i < j < k\}$
 (c) Draw a Turing Machine that accepts the language $\{a^n b^n a^n \mid n \geq 0\}$ over $\{a, b\}^*$. Also trace the TM on input string aaabbbbaaa. 07

OR

- Q.5** (a) Define Context Sensitive Language and Context Sensitive Grammar. Write CSG for $L = \{a^n b^n c^n \mid n \geq 1\}$. 03
 (b) Define - Primitive recursive functions and also give complete primitive recursive derivations for the function, $f : \mathbb{N} \rightarrow \mathbb{N}$ defined by $\text{Add}(x, y) = x + y$. 04
 (c) Draw a Turing Machine that accepts the language $\{xx \mid x \in \{a, b\}^*\}$. Also trace the TM on input string aa. 07
