Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (NEW) EXAMINATION – WINTER 2019

Subject Code: 2110014 **Date: 17/01/2020**
Subject Name: Calculus
Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)** **Mark**

(a) **07**

1. The sum of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...$
   (A) 1 (B) 2 (C) 3 (D) Infinity
2. The series $\sum_{n=1}^{\infty}\frac{1}{n}$ is
   (A) convergent (B) divergent (C) Oscillating (D) none
3. The series $\sum_{n=1}^{\infty}\frac{\sin n}{n^2}$ is
   (A) convergent (B) divergent (C) Oscillating (D) none
4. The curve $9y^2 = x(x-1)^2$ is symmetric about
   (A) *x-axis* (B) *y-axis* (C) *Line y=x* (D) *origin*
5. A point (a,b) is said to be a saddle point if at (a,b)
   (A) $rt-s^2>0$ (B) $rt-s^2<0$ (C) $rt-s^2=0$ (D) $rt-s^2\le 0$
6. The volume of solid generated by revolving a circle $x^2+y^2=9$ about *x-axis*
   (A) $\frac{4\pi}{3}$ (B) 36 (C) $\frac{36\pi}{3}$ (D) 36π
7. The value of $\lim_{x\to\infty}\left(\frac{\sin x}{x}\right)$
   (A) 1 (B) 0 (C) 2 (D) Infinity

(b) **07**

1. Which of the following is homogeneous function of degree one?
   (A) $\frac{x^2}{y}$ (B) $\frac{x^2}{y^2}$ (C) $\frac{x+2}{y}$ (D) $\frac{yx}{yx+xy}$
2. The value of $\lim_{(x,y)\to(1,1)}\frac{x-y}{x^2-y^2}$
   (A) 2 (B) 0 (C) Infinity (D) $\frac{1}{2}$

3. If $x = r\cos\theta, y = r\sin\theta$ then the value of $\frac{\partial x}{\partial r}$

(A) $\cos\theta$ (B) $\sec\theta$ (C) $\operatorname{cosec}\theta$ (D) $\sin\theta$

4. The value of $\int_1^2\int_0^x \frac{\ln x}{x}\,dxdy$

(A) 2ln2-2 (B) 2ln2-1 (C) ln2 (D) 0

5. The value of $\lim_{x\to 0} x^x$

(A) 1 (B) e (C) x (D) 0

6. The value of $\int_0^1\int_0^x\int_0^y dxdydz$

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{6}$

7. The value of $\int_0^{\frac{\pi}{2}}\int_0^{\sin\theta} r^3 drd\theta$

(A) $\frac{\pi}{32}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{64}$ (D) None

Q.2 **(a)** Define Jacobian and show that $J * J' = 1$. **03**
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(b) Find the equations of tangent plane and normal line to $x^2 + y^2 + z^2 = 81$ at the point $(-1,-4,8)$ **04**

(c) A rectangular box open the top is to have a volume of 108 c.c. find the dimension of the box requiring least material for its construction. **07**

Q.3 **(a)** Show that $\frac{\partial^2\Omega}{\partial u\partial v} = \frac{\partial^2\Omega}{\partial v\partial u}$ where $\Omega = y + x^y$ **03**

(b) Discuss the continuity of $f(x,y) = \begin{cases} \frac{x^2y^2}{x^4+4y^4}; & (x,y)\neq(0,0) \\ \frac{1}{5}; & (x,y)=(0,0) \end{cases}$ **04**

(c) State and prove Euler's Theorem for Homogeneous functions. **07**

Also, if $u = \sin^{-1}\left(\frac{x^2+y^2}{x^{\frac{3}{2}}+y^{\frac{3}{2}}}\right)$ then show that

i. $xu_x + yu_y = \frac{1}{2}\tan u$

ii. $xxu_{xx} + 2xyu_{xy} + yyu_{yy} = \frac{1}{4}\left(\tan^3 u - \tan u\right)$

Q.4 **(a)** Evaluate $\iint_A r\sin\theta\, dr\, d\theta$ over the area of the curve $r = \frac{(1+\cos\theta)}{2}$ above the initial line. **03**

(b) Evaluate the integral by the changing the order of integration, **04**

$$\int_0^8 \int_{\sqrt[3]{y}}^2 \sqrt{x^4+1}\, dx\, dy$$

(c) i. Use triple integral to find the volume of the cylinder $x^2+y^2=1$ between the planes $z=1$ and $z=2$. **03**

ii. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ by changing to polar coordinates **04**

Q.5 **(a)** Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$, if convergent then find its value. **03**

(b) Test the convergence of the series **04**

$$\frac{1}{1*2*3}+\frac{3}{2*3*4}+\frac{5}{3*4*5}+\cdots,$$

(c) For which value of x does the series $\frac{x^2}{2}-\frac{x^3}{3}+\frac{x^4}{4}-\frac{x^5}{5}+\ldots$ is absolute or conditionally convergent or divergent? What is the radius of convergent of $\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{x^5}{5}+\ldots$? **07**

Q.6 **(a)** Determine the convergent of $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{1+n^2}$ **03**

(b) Find the volume of the solid generated by revolving the region bounded by $x=y^2$ and the lines $x=0, x=2$ about the *x-axis.* **04**

(c) Trace the curve $r=a(1+\cos\theta); a>0$. **07**

Q.7 **(a)** Expand $\sin\left(x+\frac{\pi}{4}\right)$ in powers of x by using the Taylor's series. Also, find the value of $\sin 46^\circ$. **03**

(b) Find $\lim_{x\to 0}\left(\frac{e^x+e^{2x}+e^{3x}}{3}\right)^{\frac{1}{3}}$ **04**

(c) Discuss the convergence of the following integrals: **07**

(i) $\int_{-1}^{1} \frac{1}{x^2} dx$ (ii) $\int_0^\infty e^{-x^2} dx$

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