Seat No.: ________ Enrolment No.

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III(OLD) EXAMINATION – SUMMER 2019

Subject Code: 130001 **Date:30/05/2019**

Subject Name: Mathematics-III

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Obtain series solution of $\frac{d^2y}{dx^2}+y=0$. **07**

(b) **Attempt any two of the following.** **07**

1) $\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y+y\cos y}$

2) $xdy-ydx=\sqrt{x^2+y^2}$

3) $\frac{dy}{dx}+y\cot x=\cos x$

4) $\sin(y-xp)=p$. Where, $p=\frac{dy}{dx}$.

Q.2 **(a)** Obtain the Frobenius series solution of $2x^2y''+3xy'-(x^2+1)y=0$. **07**

(b) **Attempt any two of the following.** **07**

1) $\frac{d^4y}{dx^4}+13\frac{d^2y}{dx^2}+36y=0$

2) $(D^2+3D+2)y=5$.

3) $y''-2y'+y=\cos 2x$

4) Solve by Method of variation of parameters. $y''+a^2y=\tan ax$.

OR

(b) **Attempt any two of the following.** **07**

1) $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log x$.

2) Using method of undetermined multipliers solve $y''+4y=8x^2$.

3) Using method of undetermined multipliers solve $y''-3y'+2y=e^x$.

4) Prove that $\int_0^\infty \frac{x^c}{c^x}dx=\frac{\Gamma(c+1)}{(\log c)^{c+1}}$

Q.3 **(a)** Define Laplace Transformation of a function $f(t)$ and using it obtain $L(\sin at)$ and $L(t^n)$. **07**

(b) **Attempt any two of the following.** **07**

1) Find the Laplace transform of $t^3+\sin 2t+5\cosh t$.

2) Evaluate $L(\sin t\sin 2t\sin 3t)$.

3) Evaluate $L[e^{-3t}(\cos 4t+3\sin 4t)]$.

4) Evaluate $L^{-1}\left[\frac{1}{(s-1)(s^2-1)}\right]$.

OR

Q.3 **(a)** Define periodic function and obtain the Laplace transformation of periodic function having fundamental period p, **07**

(b) **Attempt any two of the following.** **07**

1) Evaluate $L\left(\dfrac{1-\cos 2t}{t}\right)$.

2) Evaluate $L\left(t^3 e^{-3t}\right)$.

3) Using convolution theorem evaluate $L^{-1}\left[\dfrac{1}{s^2\left(s^2+a^2\right)}\right]$.

4) Using Laplace transform technique solve the following IVP.
$y'' + 4y = \sin t \,|\, y(0)=1, y'(0)=0$.

Q.4 **(a)** Obtain Fourier series for the function $f(x)=x-x^2$ over $-\pi<x<\pi$ and hence show that $\sum_{n=1}^{\infty}\dfrac{1}{n^2}=\dfrac{\pi^2}{12}$. **07**

(b) **Attempt any one of the following.** **07**

1) Obtain the Fourier series for the function $f(x)=x^2, -\pi<x<\pi$. Hence show that $\sum_{n=1}^{\infty}\dfrac{1}{n^2}=\dfrac{\pi^2}{6}$.

2) Find the Fourier series to represent the function $f(x)$ given by $f(x)=\begin{cases} x & \text{for } 0\le x\le \pi. \\ 2\pi - x & \text{for } \pi \le x \le 2\pi. \end{cases}$ Hence show that $\sum_{n=1}^{\infty}\dfrac{1}{(2n-1)^2}=\dfrac{\pi^2}{8}$.

3) Obtain half-range cosine series for the function $f(x)=\begin{cases} x & \text{for } 0<x<\pi/2. \\ \pi - x & \text{for } \pi/2 < x < \pi. \end{cases}$

OR

Q.4 **(a)** Expand $f(x)=e^{-x}$ as a Fourier series in the interval $(-l,l)$. **07**

(b) **Attempt any two of the following.** **07**

1) Express the following function as Fourier integral $f(x)=\begin{cases} 1 & \text{for } |x|\le 1 \\ 0 & \text{for } |x|>1 \end{cases}$,
Hence evaluate a) $\int_0^{\infty}\dfrac{\sin\omega\cos\omega x}{\omega}\,d\omega$ and b) $\int_0^{\infty}\dfrac{\sin x}{x}\,dx$.

2) Show that $\dfrac{d}{dx}\left\{x^n J_n(x)\right\}=x^n J_{-n}(x)$.

3) Show that $(2n+1)xP_n(x)=(n+1)P_{n+1}(x)+nP_{n-1}(x)$.

Q.5 **(a)** Solve the one-dimensional wave equation together with following initial & boundary conditions. **07**

$\dfrac{\partial^2 u}{\partial t^2}=c^2\dfrac{\partial^2 u}{\partial x^2}$. Where, $c^2 = T/\rho$.
$u(0,t)=u(l,t)=0, \forall\, t>0$
$u(x,0)=f(x)$ and $u_t(x,0)=g(x), \forall\, 0<x<l$

(b) **Attempt any two of the following.** **07**

1) $xp+yq=3z$
2) $(mz-ny)p+(nx-lz)q=(ly-mx)$
3) $\left(x^2-y^2-z^2\right)p+2xyq=2xz$

OR

Q.5 **(a)** A homogenous rod of conducting material of length 100 cm. has its ends kept at zero temperature and the temperature initially is **07**

$u(x,0)=\begin{cases} x & \text{for } 0\le x\le 50 \\ 100-x & \text{for } 50\le x\le 100 \end{cases}$. Find the temperature $u(x,t)$ at any time t, at a distance x.

(b) **Attempt any two of the following.** **07**

1) $z = px + qy + \sqrt{1+p^2+q^2}$

2) $p^2 - q^2 = x - y$

3) $z = p^2 + q^2$

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