Seat No.: ________ Enrolment No. ________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(NEW) – EXAMINATION – SUMMER 2019

Subject Code:2140001 **Date:09/05/2019**

Subject Name: Mathematics-4

Time:02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	Find the principal argument of $z = \frac{i}{\sqrt{3}+i}$	**03**
	(b)	Check whether the following functions are analytic or not at any point: (i) $f(z) = x^2 + ixy$ (ii) $f(z) = z^2$	**04**
	(c)	(i) Expand $f(z) = zcos\left(\frac{1}{z^3}\right)$ in Laurent's series near $z = 0$ and identify the singularity. (ii) Show that if c is any n^{th} root of unity other than unity itself, than $1 + c + c^2 + \cdots + c^{n-1} = 0$.	**07**
Q.2	**(a)**	Find and sketch the image of the region $\|z\| < 1$ under the transformation $2z - i$.	**03**
	(b)	Show that the function $u(x, y) = y^3 - 3x^2y$ is harmonic in some domain D and find its conjugate $v(x, y)$.	**04**
	(c)	Find the Mobius transformation that maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $\|z\| = 1$.	**07**
		OR	
	(c)	Evaluate the integral $\int_C \text{Re}(z^2)dz$, where C is the boundary of the square with vertices $0, i, 1 + i, 1$ in clockwise direction.	**07**
Q.3	**(a)**	Evaluate $\int_0^{1+i} (x^2 + iy)dz$ along the path $y = x^2$.	**03**
	(b)	Find the residue at each pole of $f(z) = \frac{ze^{iz}}{z^2+9}$	**04**
	(c)	Expand $f(z) = \frac{1}{(z+1)(z-2)}$ in Laurent's series in the region (i) $\|z\| < 1$ (ii) $1 < \|z\| < 2$ (iii) $\|z\| > 2$.	**07**
		OR	
Q.3	**(a)**	Write the Cauchy integral formula and using it evaluate $\int_C \frac{\cos z}{z+\pi}dz$ where C is the circle $\|z\| = 4$.	**03**
	(b)	Evaluate $\oint_C \frac{2z-1}{z(z+1)(z-3)}dz$, where C is the circle $\|z\| = 2$.	**04**
	(c)	Using the residue theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\sin\theta}$	**07**

Q.4	**(a)**	Find the positive root of the equation $2\sin x - x = 0$ using bisection method in six stages.	**03**
	(b)	Solve the following system of equations by Gauss Seidel method: $28x+4y-z=32 \quad 2x+17y+4z=35 \quad x+3y+10z=24$ Correct up to two decimal places.	**04**
	(c)	Using the power method find the largest eigenvalue of the matrix $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$	**07**

OR

Q.4	**(a)**	Use the secant method in three stages to find the root of the equation $\cos x - xe^x = 0$.	**03**
	(b)	Find an approximate value of $f(3.6)$ using Newton's backward difference formula from the following data:	**04**

x	0	1	2	3	4
f(x)	-5	1	9	25	55

	(c)	Using Lagrange's interpolation formula find *y* when *x* = 5 from the following table:	**07**

x	1	2	3	4	7
y	2	4	8	16	128

Q.5	**(a)**	Use Simpson's 1/3 rule to evaluate $\int_1^2 e^{-\frac{x}{2}} dx$. Take h = 0.25.	**03**
	(b)	Use Gauss elimination method to solve the system of equations $2x_1+4x_2-6x_3=-4; \quad x_1+5x_2+3x_3=10; \quad x_1+3x_2+2x_3=5.$	**04**
	(c)	Derive Euler's formula to solve the initial value problem $\frac{dy}{dx}=f(x,y);\ y(x_0)=y_0$. Find $y(0.1)$ for $\frac{dy}{dx}=x^2+y$, where $y(0)=1$ using improved Euler's method. Take $h=0.05$.	**07**

OR

Q.5	**(a)**	Find the real root of the equation $x^3 - 9x + 1 = 0$ up to five decimal places by the Newton-Raphson's method. Take $x_0 = 3$.	**03**
	(b)	Find $f(15)$ from the following table using Newton's divided difference formula:	**04**

x	4	5	7	10	11	13
f(x)	48	100	204	900	1210	2028

	(c)	Apply fourth order Runge-Kutta method to find $y(0.1)$ and $y(0.2)$ for the differential equation $\frac{dy}{dx}=3x+\frac{1}{2}y,\ y(0)=1$. Take $h=0.1$.	**07**

****