Seat No.: _____ Enrolment No. _____


# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER–IV(NEW) – EXAMINATION – SUMMER 2019**

**Subject Code:2140105** **Date:09/05/2019**

**Subject Name: Numerical Methods**

**Time:02:30 PM TO 05:00 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

| | | | MARKS |
|---|---|---|---|
| **Q.1** | **(a)** | Name two interpolation methods used for unequal intervals. Also state their formulas. | **03** |
| | **(b)** | Perform four iterations to find a root of the equation $x^3-4x-9=0$ using Bisection method. | **04** |
| | **(c)** | Using fourth order Runge Kutta method, find $y(0.1)$ for differential equation $\frac{dy}{dx}=2x+y, y(0)=1$ by taking h= 0.1 | **07** |
| **Q.2** | **(a)** | Solve the following system by Gauss elimination method. $x+3y+2z=5,\ 2x+4y-6z=-4,\ x+5y+3z=10$ | **03** |
| | **(b)** | Find a real root of the equation $3x=\cos x+1$, correct up to four decimal places using Newton Raphson method. | **04** |
| | **(c)** | Fit a second degree polynomial using least square method to the following data: | **07** |

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 5 | 12 | 26 | 60 | 97 |

Also estimate $y$ at $x=6$.

**OR**

| | | | | | |
|---|---|---|---|---|---|
| | **(c)** | Fit a curve of the form $y=ae^{bx}$ to the following data: | | | **07** |

| $x$ | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| $y$ | 115 | 105 | 95 | 85 | 80 |

| | | | | | |
|---|---|---|---|---|---|
| **Q.3** | **(a)** | Using Newton's forward interpolation formula, find the value of $f(1.6)$ . | | | **03** |

| $x$ | 1 | 1.4 | 1.8 | 2.2 |
|---|---|---|---|---|
| $f(x)$ | 3.49 | 4.82 | 5.96 | 6.5 |

| | | | | | |
|---|---|---|---|---|---|
| | **(b)** | Use trapezoidal rule to evaluate $\int_0^2 \frac{x}{\sqrt{2+x^2}}dx$, dividing the interval into four equal parts. | | | **04** |
| | **(c)** | Use Gauss-Siedel method to solve the following system: $6x+y+z=105,\ 4x+8y+3z=155,\ 5x+4y-10z=65$ | | | **07** |

**OR**

**Q.3** **(a)** Evaluate $f(9)$ by using Lagrange's interpolation method from the following data: **03**

| $x$ | 5 | 7 | 11 | 13 | 17 |
|---|---|---|---|---|---|
| $f(x)$ | 150 | 392 | 1452 | 2366 | 5202 |

**(b)** Evaluate $\int_0^3 \frac{1}{1+x}dx$ with $n=6$ by using Simpson's $3/8$ rule. **04**

**(c)** Compute $y(1.5)$ & $y'(1)$ from the following data using Cubic Spline. **07**

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $y$ | -8 | -1 | 18 |

**Q.4** **(a)** Use Taylor's series method to find $y$ at $x=0.03$ given that $\frac{dy}{dx}=x^2y-1,\ y(0)=1.$ **03**

**(b)** Find the root of $x\log_{10}x-1.9=0,$ correct up to three decimal places with $x_0=3$ *and* $x_1=4$ using Secant method. **04**

**(c)** Using Shooting method , Solve the boundary value problem: **07**
$y''=y,\ y(0)=0$ *and* $y(1)=1.17$

**OR**

**Q.4** **(a)** Solve the following system by Gauss Jordan method: **03**
$x-2y=-4,\ -5y+z=-9,\ 4x-3z=-10$

**(b)** Solve the equation $y''=x+y$ with the boundary conditions $y(0)=y(1)=0$ by finite difference method. **04**

**(c)** Using Picard's method of successive approximation, obtain a solution up to fifth approximation of the equation $\frac{dy}{dx}=x+y, y(0)=1.$ **07**

**Q.5** **(a)** Explain Initial value problem and boundary value problem with example. **03**

**(b)** Solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ in $0<x<5,\ t\geq 0$ given that $u(x,0)=20,\ u(0,t)=0,\ u(5,t)=100$ . Compute $u(x,t)$ with h=1 by Crank-Nicholson method. **04**

**(c)** Solve the boundary value problem $y''-x=0$ , $y(0)=0$ *and* $y'(1)=-\frac{1}{2}$ by the Rayleigh-Ritz method. **07**

**OR**

**Q.5** **(a)** State the difference between finite difference method and finite element method. **03**

**(b)** Discuss the concept of Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ **04**

**(c)** Solve the boundary value problem $y''+y=-x,\ \ y(0)=0,\ y(1)=0$ by the Galerkin method. **07**

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