FirstRanker.com
Firstranker's choice

Seat No.: ________ www.FirstRanker.com Enrolment No. www.FirstRanker.com

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (SPFU) EXAMINATION – SUMMER-2019

Subject Code: MTH001 **Date: 07/06/2019**

Subject Name: Calculus

Time: 10:30 AM TO 01:00 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{(n+1)^n x^n}{n^{n+1}}$ **04**

(ii) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1} n}{2n-1}$ **03**

(b) If $u=r^m, r^2=x^2+y^2+z^2$ show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=m(m+1)r^{m-2}$ **07**

Q.2 **(a)** (i) Show that $f(x,y)=\frac{xy}{\sqrt{x^2+y^2}}, \quad (x,y)\neq(0,0)$
$=0, \quad (x,y)=(0,0)$ is continuous at the origin. **04**

(ii) If $x=r\cos\theta, y=r\sin\theta$, show that $\left(\frac{\partial r}{\partial x}\right)^2+\left(\frac{\partial r}{\partial y}\right)^2=1.$ **03**

(b) Determine absolute or conditional convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{n^3+1}$ **07**

Q.3 **(a)** (i) Evaluate $\int_0^1\int_0^x dydx$. **03**

(ii) Evaluate $\iint xydxdy$ over the region enclosed by the x-axis, the line $x=2a$ and the parabola $x^2=4ay$. **04**

(b) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that **07**

(i) $2x\frac{\partial u}{\partial x}+2y\frac{\partial u}{\partial y}=\tan u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{1}{4}\left(\tan^3 u-\tan u\right)$

Q.4 **(a)** (i) Find the equation of the tangent plane and normal line to the surface $z=2x^2+y^2$ at the point (1, 1 ,3). **04**

(ii) If $u=y^2-4ax, x=at^2, y=2at$ find $\frac{du}{dt}$. **03**

(b) Use triple integral to find the volume of the solid within the cylinder $x^2+y^2=9$ between the planes $z=1$ and $x+z=1$. **07**

www.FirstRanker.com

www.FirstRanker.com

FirstRanker.com
Firstranker's choice
www.FirstRanker.com www.FirstRanker.com

Q.5 **(a)** (i) Prove that $1+\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\frac{16}{81}+........$ converges and find its term. **03**

(ii)Investigate the convergence of the series $\sum_{n=1}^{\infty}\frac{2^n+5}{3^n}$ **04**

(b) If $u=f(r)$ and $r^2=x^2+y^2+z^2$, prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f''(r)+\frac{2}{r}f'(r)$ **07**

Q.6 **(a)** (i) Discuss the maxima and minima of the function $3x^2-y^2+x^3$ **04**

(ii)Test the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$ **03**

(b) Evaluate $\iiint xyz\,dxdydz$ over the positive octant of the sphere $x^2+y^2+z^2=4$. **07**

Q.7 **(a)** Evaluate $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx\,dy$ by changing the order of integration. **07**

(b) (i) Find the minimum value of x^2+y^2, subject to the condition $ax+by=c$. **04**

(ii)Expand e^{x+y} in power of $(x-1)$ and $(y+1)$ up to first degree terms. **03**

****

www.FirstRanker.com

www.FirstRanker.com