Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (OLD) EXAMINATION – SUMMER-2019

Subject Code: 110009 **Date: 01/06/2019**

Subject Name: Maths - II

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q-1 **(a)** (i) Find the value of λ so that the equations $\begin{matrix} 2x+y+2z=0 \\ x+y+3z=0 \\ 4x+3y+\lambda z=0 \end{matrix}$; have a non trivial solution. **4**

(ii) Verify Cauchy-Schwarz inequality for the vectors $\bar{u}=(-3,1,0)$, $\bar{v}=(2,-1,3)$. **3**

(b) Solve the following equations by Gauss elimination and back substitution, **7**

$x+y+2z=9$
$2x+4y-3z=1$
$3x+6y-5z=0$

Q-2 **(a)** (i) Obtain the reduced row echelon form of the matrix $\begin{bmatrix} 1 & 3 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 8 \end{bmatrix}$. **4**

(ii) Find the rank of the matrix, if $A=\begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$. **3**

(b) Use row operation to find A^{-1}, if $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$. **7**

Q-3 **(a)** (i) Find the eigen values and eigen vectors of the matrix, $A=\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **4**

(ii) Using Cayley-Hamilton theorem, find A^2, if $A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. **3**

(b) Find a matrix P that diagonalizes $A=\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ and hence find A^{10}. **7**

Q-4 **(a)** (i) Reduce $S=\{(1,0,0),(0,1,-1),(0,4,-3),(0,2,0)\}$ to obtain a basis for R^3. **4**

(ii) Determine whether or not the vectors $\{(1,2,2),(2,1,2),(2,2,1)\}$ in R^3 are linearly independent. **3**

(b) Let $V=\{(x,y) \mid x,y\in R, y>0\}$, let $(a,b),(c,d)\in V, \alpha\in R$.
Define $(a,b)+(c,d)=(a+c, b\cdot d)$ and $\alpha\cdot(a,b)=(\alpha a, b^{\alpha})$. Prove that V is a vector space. **7**

Q-5 **(a)** (i) Show that the transformation $T:R^2\to R^2$, where $T(x,y)=(2x-y, x-y)$ is a linear transformation. **4**

(ii) Express the quadratic form $Q(x,y)=2x^2+3y^2+6xy$, in matrix notation. **3**

(b) Find the rank and nullity of the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ **7**

Q-6 **(a)** (i) Find a basis for the orthogonal complement of the subset of R^3 spanned by the vectors $\bar{v}_1 = (1,-1,3), \bar{v}_2 = (5,-4,-4), \bar{v}_3 = (7,-6,2)$. **4**

(ii) Determine whether the linear transformation $T: R^2 \to R^3$, where $T(x, y) = (x, y, x + y)$ is one-one. **3**

(b) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $S = \{(1,1,1), (-1,1,0), (1,2,1)\}$ into an Orthonormal basis. **7**

Q-7 **(a)** (i) Prove that $A = \begin{bmatrix} 1 & 3+4i & -2i \\ 3-4i & 2 & 9-7i \\ 2i & 9+7i & 3 \end{bmatrix}$ is a Hermitian matrix. **4**

(ii) Let R^2 have the Euclidean inner product. Find the cosine of the angle θ between the vectors $\bar{u} = (4,3,1,-2)$ and $\bar{v} = (-2,1,2,3)$. **3**

(b) Find the least square solution of the linear system $AX = b$ and find the orthogonal projection of b onto the column space of A, where $A = \begin{bmatrix} 2 & -2 \\ 1 & 1 \\ 3 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ **7**

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