Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (OLD) EXAMINATION – SUMMER-2019

Subject Code: 110014 **Date: 06/06/2019**

Subject Name: Calculus

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (i) State Euler's theorem on homogeneous function. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, then prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u \cos 3u$ **05**

(b) (i) If $u = xy^2 + y^3 + x^3 + z^3$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3u$ **03**

(ii) If $u = \ln\left(x^3 + y^3 + z^3 - 3xyz\right)$, prove that **03**

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x + y + z)^2}$$

(c) Determine whether $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$ exist or not? If they exist find the value of the limit. **03**

Q.2 (a) Find maxima and minima of the function **05**

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$

(b) Expand $e^x \tan^{-1} y$ about $(1,1)$ up to second degree in $(x-1)$ and $(y-1)$. **05**

(c) If $x = r\cos\theta, y = r\sin\theta$, find $\frac{\partial(x, y)}{\partial(r, \theta)}$ and $\frac{\partial(r, \theta)}{\partial(x, y)}$ **04**

Q.3	**(a)**		Expand $\frac{e^x}{\cos x}$ in Maclaurin's series.	**05**
	(b)	**(i)**	Evaluate $\lim_{x\to 0} \frac{\tan x - \sin x}{x^2}$.	**02**
		(ii)	Find the values of a and b such that, $\lim_{x\to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$	**03**
	(c)		Using Taylor's series find $\sqrt[3]{27.12}$ correct to four decimal places.	**04**
Q.4	**(a)**		Trace the curve $y^2(a+x) = x^2(3a-x)$	**05**
	(b)		Trace the curve $r = a(1+\cos\theta)$	**05**
	(c)		Using reduction formula evaluate (i) $\int_0^{\pi/2} \cos^5 dx$ and (ii) $\int_0^{\pi} \sin^6 dx$	**04**
Q.5	**(a)**	**(i)**	Test the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$	**02**
		(ii)	Test the convergence of $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$	**03**
	(b)		Test the convergence of $\sum_{n=1}^{\infty} ne^{-n^2}$	**04**
	(c)		Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x dx$	**05**
Q.6	**(a)**		Evaluate $\iint_R \sin\theta \, dA$, where R the region is in the 1st quadrant. i.e. outside the circle $r=2$ and inside the cardioids $r = 2(1+\cos\theta)$.	**05**
	(b)		Evaluate by changing the order of integration $\int_0^1 \int_{x^2}^{2-x} xy \, dy dx$	**05**
	(c)		Find the volume bounded by cylinder $x^2+y^2=4$ and the planes $y+z=4$, $z=0$.	**04**

Q.7	**(a)**	Prove that $\int_1^{\infty} \frac{1}{x^p} dx$, converges when $p > 1$ and diverges when $p \le 1$	**05**
	(b)	Use triple integral in cylindrical co-ordinate to find the volume of solid, bounded above the hemisphere $z = \sqrt{25 - x^2 - y^2}$, below by $xy - plane$ and laterally by the cylinder $x^2 + y^2 = 9$	**05**
	(c)	Find the volume of a cone with height $4cm$ and radius of base $4cm$. Use the method of slicing.	**04**

****