Seat No.: ________ Enrolment No.

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – SUMMER 2019

Subject Code: 2130002 **Date: 30/05/2019**

Subject Name:Advanced Engineering Mathematics

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1	**(a)**	Solve $(x+y-2)\,dx+(x-y+4)\,dy=0$	**03**
	(b)	Solve $(1+y^2)+(x-e^{-\tan^{-1}y})\dfrac{dx}{dy}=0$	**04**
	(c)	Expand $f(x)=\lvert\cos x\rvert$ as a Fourier series in the interval $-\pi<x<\pi$	**07**
Q.2	**(a)**	Define unit step function and unit impulse function. Also sketch the graphs.	**03**
	(b)	Solve $\left(\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y\right)=4\sin 2x$	**04**
	(c)	Find the series solution of $y''+xy'+y=0$ about the ordinary point $x=0$.	**07**
		OR	
	(c)	Find the Fourier series expansion for $f(x)$, if $f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$, Also deduce that $1+\dfrac{1}{3^2}+\dfrac{1}{5^2}+......=\dfrac{\pi^2}{8}$	**07**
Q.3	**(a)**	Using Fourier integral representation, show that $\displaystyle\int_0^\infty \frac{1-\cos\pi\omega}{\omega}\sin\omega x\,d\omega=\begin{cases}\dfrac{\pi}{2}, & 0<x<\pi\\ 0, & x>\pi\end{cases}$	**03**
	(b)	Solve $\left(\dfrac{d^2y}{dx^2}+y\right)=x^2\sin 2x$	**04**
	(c)	Solve by method of variation of parameters $\left(\dfrac{d^2y}{dx^2}+9y\right)=\dfrac{1}{1+\sin 3x}$	**07**
		OR	
Q.3	**(a)**	Find Laplace transform of $te^{at}\sin at$	**03**
	(b)	Solve $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=5e^x-\sin 2x$	**04**

(c) Solve

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$$ 07

Q.4 (a) Find the orthogonal trajectories of the curve $y = x^2 + c$ 03

(b) Find the Laplace transform of (i) $\cos(at+b)$ 04

(ii) $\sin^2 3t$

(c) State convolution theorem and apply it to evaluate 07

$$L^{-1}\left(\frac{s^2}{(s^2+4)^2}\right)$$

OR

Q.4

(a) Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$ 03

(b) Find Half range cosine series for $f(x) = (x-1)^2$ in the interval $0 < x < 1$ 04

(c) Solve $y'' + 4y' + 3y = e^{-t}, y(0) = y'(0) = 1$ using Laplace transform. 07

Q.5 (a) Form the partial differential equation by eliminating the arbitrary constants from $z = ax + by + a^2 + b^2$ 03

(b) Solve $(y-z)p + (x-y)q = z - x$ 04

(c) Solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,$ where $u(x,0) = 4e^{-x}$ using the method of separation of variables. 07

OR

Q.5 (a) Form the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2, z - xy) = 0$ 03

(b) Solve $\log\left(\frac{\partial^2 z}{\partial x \partial y}\right) = x + y.$ 04

(c) A bar with insulated sides is initially at temperature $0^0 C$, throughout. The end $x = 0$ is kept at $0^0 C$ and heat is suddenly applied at the end $x = l$ so that $\frac{\partial u}{\partial x} = A$ for $x = l$, where A is a constant. Find the temperature function. 07

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