Seat No.: ________ Enrolment No.

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (OLD) EXAMINATION – WINTER 2018

Subject Code:140001 **Date: 22/11/2018**

Subject Name: Mathematics-IV

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Define harmonic function and show that $f(x,y) = x^2 - y^2$ is harmonic. **03**

(ii) Show that $lim_{z\to 0}\frac{\bar{z}}{z}$ does not exist. **04**

(b) Apply Gauss-Seidel method to solve the equations $20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25.$ **07**

Q.2 **(a)** (i) State Trapezoidal rule, Simpson's 1/3rd rule and Simpson 3/8th rule. **03**

(ii) Find the positive root of $x^3 - x - 11 = 0$ correct to the three decimal places by Bisection method. **04**

(b) Obtain approximate value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x,\ y(0) = 0$, using Taylor's method. Also compare the obtained numerical solution with the exact solution. **07**

OR

(b) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at $x = 0.2$. **07**

Q.3 **(a)** (i) Evaluate $\int_0^1 e^{-x^2}\,dx$ using Gauss quadrature formula of three points. **03**

(ii) Using usual notations, show that $\Delta= E\Delta= E\nabla= \delta E^{1/2}$ and $(E^{1/2} + E^{-1/2})\,(1+\Delta)^{1/2} = 2+\Delta$. **04**

(b) Using Lagrange's interpolation formula, find the value of y when $x = 10$, if the following values of x and y are given: **07**

x	5	6	9	11
y	12	13	14	16

OR

Q.3 **(a)** (i) Define entire function and show that $f(z) = e^z$ is entire function. **03**

(ii) Find harmonic conjugate of $u(x,y) = 2x - x^3 + 3xy^2$. **04**

(b) Using Newton forward interpolation formula, find the value of $f(1.6)$ if the following values of x and $f(x)$ are given: **07**

x	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.96	6.5

Q.4 **(a)** (i) Evaluate $\oint_c \frac{e^{-z}}{z+1}dz$, where c is a circle $|z| = 1/2$. **03**

(ii) Determine and sketch the image of $|z| = 1$ under the transformation$w = z + i$. **04**

(b) Find Maclaurin's series expansion of the function $f(z) = sin^2 z$. **07**

OR

Q.4	**(a)**	(i) Evaluate $\oint_c \frac{e^z}{z+1} dz$, where c is a circle $\lvert z\rvert = 2$.	**03**
		(ii) Find bilinear transformation that maps the points -1, 0, 1 onto –1, -i,1 respectively.	**04**
	(b)	Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series valid for $\lvert z\rvert < 1$ and $1 < \lvert z\rvert < 3$.	**07**
Q.5	**(a)**	(i) Find an upper bound for the absolute value of the integral $\int_C e^z dz$, where C is the line segment joining the points (0,0) and (1, $2\sqrt{2}$).	**03**
		(ii) Determine the poles of the function $f(z) = \frac{1}{(z^2+1)^2}$ and the residue at each poles.	**04**
	(b)	Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)\,(x^2+2x+2)}$.	**07**

OR

Q.5	**(a)**	(i) State (i) Rouche's Thorem (ii) Liouville's Theorem	**03**
		(ii) Evaluate $\oint_C \frac{e^{3z}}{(z-ln2)^4} dz$, where C is the square with vertices at $\pm1, \pm i$.	**04**
	(b)	Evaluate $\int_0^{2\pi} \frac{d\theta}{(2+cos\theta)^2}$.	**07**

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