Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (OLD) EXAMINATION – WINTER 2018

Subject Code:130001 **Date:17/11/2018**

Subject Name:Mathematics-III

Time:10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (1) Solve $x(x-1)\frac{dy}{dx}-(x-2)=x^3(2x-1)$ **03**

(2) Solve $(x^2y^2+2)y\,dx+(2-x^2y^2)x\,dy=0$ **04**

(b) Find the power series solution of the equation $4x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$ about $x=0$. **07**

Q.2 **(a)** (1) Solve $(D^2-4)y=e^x+\sin 2x$. **03**

(2) Solve $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{e^x}{1+e^x}$ by using method of variation of parameter. **04**

(b) Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$ **07**

OR

(b) Find series solution of the differential equation $(1+x^2)\,y''+xy'-y=0$. **07**

Q.3

(a) (1) Show that $\int_0^1 x^2(1-x)^3\,dx=\frac{1}{60}$. **03**

(2) Prove that $\frac{d}{dx}\left(x^nJ_n(x)\right)=x^nJ_{n-1(x)}$ **04**

(b) Find the half range cosine series for $f(x)=\begin{cases}kx & ;\ 0\le x\le l/2\\ k(l-x) & ;\ l/2\le x\le l\end{cases}$

Also Prove that $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$ **07**

OR

Q.3 **(a)** Find the Fourier series of $f(x)=x\sin x$ in the interval $(-\pi,\pi)$. Hence, deduce that $\frac{\pi-1}{4}=\frac{1}{1\cdot 3}-\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}-\dots$ **07**

(b) (1) Evaluate $\int_0^1 x^5(\log x)^5\,dx$ **03**

(2) Find half range Fourier sine series of the function $f(x)=\pi-x$ for $0<x<\pi$. **04**

Q.4 **(a)** (1) Find the Laplace transform of the function $f(t)=\sin\sqrt{t}$. **03**

(2) Find the inverse Laplace transform of the function $F(s) = \dfrac{6s-4}{s^2-4s+20}$. **04**

(b) Solve the differential equation using Laplace Transformation method

$\dfrac{d^2y}{dt^2} + y = t\cos t$, Given that $y(0) = 0$, $y'(0) = 0$, $t > 0$. **07**

OR

Q.4 **(a)** (1) Find the Laplace transform of the function $f(t) = t\cos t$ **03**

(2) Find the inverse Laplace transform of the function $F(s) = \log\left(1 + \dfrac{1}{s^2}\right)$ **04**

(b) Define Convolution theorem for Laplace transform. Using Convolution theorem to find Laplace inverse of the function $F(s) = \dfrac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$ **07**

Q.5 **(a)** (1) Form the partial differential equation of $f\left(xy + z^2,\ x + y + z\right) = 0$. **03**

(2) Solve $(y+z)\,p + (x+z)\,q = x + y$. **04**

(b) Solve by the method of separation of variables $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 0$ **07**

OR

Q.5 **(a)** Using method of separation of variables, solve $4\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 3u$, given that $u(0, y) = 3e^{-y} - e^{-5y}$. **07**

(b) (1) Solve $\left(x^2 + y^2 + 1\right)dx - 2xy\,dy = 0$ **03**

(2) Solve $(D^3 + 3D^2 + 2D)y = x^2 + 4x + 8$ by using method of undetermined coefficients. **04**

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