Seat No.: _____  Enrolment No. 

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER–IV (NEW) EXAMINATION – WINTER 2018**

**Subject Code:2140001** **Date:22/11/2018**

**Subject Name:Mathematics-4**

**Time: 02:30 PM TO 05:30 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

| | | | |
|---|---|---|---|
| **Q.1** | **(a)** | Find the complex conjugate of $\frac{5+2i}{1-i}$ | **03** |
| | **(b)** | Find the locus of z given by $\left\|\frac{z-1}{z+1}\right\| = 1$. | **04** |
| | **(c)** | Show that $u = y^3 - 3x^2y$ is a harmonic function. Also find its harmonic conjugate. | **07** |
| **Q.2** | **(a)** | Determine the region in the z-plane represented by $1 < \|z-2\| < 3$. | **03** |
| | **(b)** | Show that $\frac{1+2z}{z^2+z^3} = \frac{1}{z^2} + \frac{1}{z} - 1 + z - z^2 + \cdots in\ 0 < \|z\| < 1$. | **04** |
| | **(c)** | Find the roots common to the equation $z^4 + 1 = 0\ and\ z^6 - i = 0$. | **07** |
| | | **OR** | |
| | **(c)** | Evaluate $\int_C \bar{z}dz$ along the straight line joining $z = 1 - i\ to\ z = 3 + 2i$. | **07** |
| **Q.3** | **(a)** | Expand $f(z) = \frac{1}{z}$ as a Taylor's series about the point $z_0 = 1$. Also determine the region of convergence and radius of convergence. | **03** |
| | **(b)** | Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. | **04** |
| | **(c)** | Evaluate $\int_0^{2\pi} \frac{cos2\theta}{5+4cos\theta} d\theta$ | **07** |
| | | **OR** | |
| **Q.3** | **(a)** | Determine and sketch the image of $\|z\| = 1$ under the transformation $w = z + i$. | **03** |
| | **(b)** | Determine the poles of the equation $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and residue at each pole. | **04** |
| | **(c)** | Evaluate $\int_C Re(z^2)dz$ , where C is the boundary of the square with vertices $0, i, 1+i, 1$ in the clockwise direction. | **07** |

**Q.4** **(a)** Using Simpson's $\frac{1}{3}$ rule evaluate $\int_1^{2.5} f(x)\,dx$ from the data given below. **03**

| x | 1 | 1.3 | 1.6 | 1.9 | 2.2 | 2.5 |
|---|---|---|---|---|---|---|
| F(x) | 1 | 1.69 | 2.56 | 3.61 | 4.84 | 6.25 |

**(b)** Solve the following system of equation using Gauss Elimination method with partial pivoting **04**

$$x + y + z = 7$$
$$3x + 3y + 4z = 24$$
$$2x + y + 3z = 16$$

**(c)** Find the values of $y\ for\ x = 21\ and\ x = 28$ from the following data. **07**

| x | 20 | 23 | 26 | 29 |
|---|---|---|---|---|
| y | 0.3420 | 0.3907 | 0.4384 | 0.4848 |

**OR**

**Q.4** **(a)** Find the largest eigenvalue and corresponding eigen vector for $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ **03**

**(b)** Find the positive root of $x = cosx$ correct upto 3 decimal places, using N-R method. **04**

**(c)** Solve the following system by Gauss-Jacobi method. **07**

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

**Q.5** **(a)** Evaluate $\Delta^2 cos2x$ **03**

**(b)** Express the function $\frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}$ as a sum of partial fraction, using Largrange's formula. **04**

**(c)** Find the value of y for $\frac{dy}{dx} = x + y; y(0) = 1, when\ x = 0.1, 0.2$ with step size h =0.05. Also compare with analytic solution. **07**

**OR**

**Q.5** **(a)** Find a root of the equation $x^3 - x - 11 = 0$, using the bisection method up to fourth approximation. **03**

**(b)** From the following table, find $f(x)$ using Newton's divided difference formula **04**

| x | 1 | 2 | 7 | 8 |
|---|---|---|---|---|
| f(x) | 1 | 5 | 5 | 4 |

**(c)** Determine the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 4 & 4 & 2 \\ 4 & 4 & 1 \\ 2 & 1 & 8 \end{bmatrix}$ **07**

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