

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-IV (NEW) EXAMINATION – WINTER 2018

Subject Code:2140105

Date:22/11/2018

Subject Name:Numerical Methods

Time: 02:30 PM TO 05:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Name five iterative methods which evaluate the root of equations. **03**
- (b) Perform five iterations of Bisection method to obtain real root of $x^3 - x - 1 = 0$. **04**
- (c) By the method of least squares, find the straight line $y = ax + b$ that best fits the following data: **07**

x	0	5	10	15	20	25
y	12	15	17	22	24	30

- Q.2** (a) Mention atleast two difference between Newton's forward Interpolation and Newton's divided difference interpolation. **03**
- (b) Find second degree polynomial passing through the points (-1, 8), (0, 3), (2, 1) and (3, 12) using Lagrange interpolation. **04**
- (c) Obtain cubic splines for every subintervals from the following data: **07**

x	0	1	2	3
y	1	2	33	244

OR

- (c) Using Newton's Divided Difference Interpolation find $f(x)$ from the following table: **07**

x	1	2	7	8
y	1	5	5	4

- Q.3** (a) Use Gauss Elimination to solve: **03**
- $$\begin{aligned} x + 3y + 2z &= 5 \\ 2x + 4y - 6z &= -4 \\ x + 5y + 3z &= 10 \end{aligned}$$

- (b) Consider following tabular values: **04**

x	25	25.1	25.2	25.3	25.4	25.5	25.6
F(x)	3.205	3.217	3.232	3.245	3.256	3.268	3.280

Determine the area bounded by given curve and x-axis between $x = 25$ to $x = 25.6$ by the Trapezoidal rule.

- (c) Describe Newton-Raphson method and find root of equation $x \sin x + \cos x = 0$ which is near π correct upto 5 decimal places. **07**

OR

- Q.3** (a) Find approximate root of $x^3 - 2x - 1 = 0$ starting from $x_0 = 1.5$ to $x_1 = 2$ by Secant method correct upto 3 decimal places. **03**

- (b) Evaluate by Simpson's $\frac{1}{3}$ Rule, $\int_0^5 \frac{1}{5+4x} dx$ taking 10 equal parts, hence obtain approximate value of $\log_e 5$. **04**

- (c) State diagonally dominant condition and evaluate by Gauss-Seidel method: 07
 $5x + y - z = 10$
 $2x + 4y + z = 14$
 $x + y + 8z = 20$
- Q.4 (a)** State finite difference quotients for first and second order derivatives. 03
- (b)** Solve heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with $u(x, 0) = 0$, $u(0, t) = 0$ and $u(1, t) = t$ with $k = \frac{1}{8}$, $h = \frac{1}{4}$. 04
- (c)** Solve by Runge-Kutta fourth order $\frac{dy}{dx} = 2x + y$, $y(0) = 1$, $h = 0.1$ find $y(0.1)$ and $y(0.2)$. 07
- OR**
- Q.4 (a)** State Gauss-Seidel method for Laplace equation. 03
- (b)** Discuss shooting approach for Boundary Value Problem in brief. 04
 Write two differences between finite difference method and finite element method. 07
- Q.5 (a)** Evaluate (1) $(1 + \Delta)(1 - \nabla) = 1$ (2) $\Delta = E\nabla$ 03
- (b)** Solve by Runge-Kutta second order $\frac{dy}{dx} = 3x + y$, $y(1) = 1.3$, $h = 0.1$ find $y(1.2)$. 04
- (c)** Evaluate IVP $\frac{dy}{dx} = x\sqrt{y}$, $y(1) = 1$ and hence find $y(1.5)$ taking $h = 0.1$ by Euler's method. 07
- OR**
- Q.5 (a)** Describe Galerkin approach in brief. 03
- (b)** Solve $y'' = x + y$ with boundary conditions $y(0) = y(1) = 0$ by finite difference method. 04
- (c)** Describe Rayleigh Ritz method in brief. 07
