

Code No. 9002 / M

FACULTY OF ENGINEERING AND INFORMATICS
B.E. I Year (Main) Examination, May / June 2016
Subject : Mathematics - I
Time : 3 hours
Max. Marks : 75
Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.
PART - A (25 Marks)

1 Let $f(x) = \frac{1}{3-x^2}$ and $f(0) = \frac{1}{3}$. Find an interval in which $f(1)$ lies. (2)

2 Find the equation of the envelope of the family of straight lines $y = cx + c^2$ where c is a parameter. (3)

3 Prove that $f(x, y) = \begin{cases} \frac{x^2 - xy + x + y}{x + y}; & (x, y) \neq (2, 2) \\ 4; & (x, y) = (2, 2) \end{cases}$ is discontinuous at the point $(2, 2)$. (2)

4 If $f(x, y) = \tan(x/y)$, find an approximate value of $f(1.1, 0.8)$ using the Taylor series linear approximation. (3)

5 Evaluate the double integral $\iint_R xy \, dx \, dy$, where R is the region bounded by the x -axis, the line $y = 2x$ and the parabola $x^2 = 4ay$. (2)

6 If a is a constant vector, and $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ then prove that $\nabla \cdot (ax\mathbf{i}) = a$. (3)

7 Test whether the vectors $(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 3)$ are linearly independent or not. (2)

8 Find all values of A for which rank of the matrix. (3)

$$A = \begin{bmatrix} \lambda & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 2 & -1 \\ -6 & 11 & 6 & 1 \end{bmatrix}$$

is equal to 3.

9 Test the convergence of the series $\sum_{n=1}^{\infty} \left| \frac{(1+nx)^n}{n^n} \right|$ (2)

10 Show by an example that every convergent series need not be absolutely convergent. (3)

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PART - B (50 Marks)

11 a) State and prove Lagrange's mean value theorem. (5)

 b) Find the evolute of $x^2 = 4ay$. (5)

 12 a) Find the shortest distance between the line $y = 10 - 2x$ and the ellipse (5)

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

 b) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$ does not exist. (5)

13 a) Show that the vector field defined by the vector function

 $xyz(yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k})$ is conservative.

 b) Show that $\int_C (yz \, dx + (z + xz + z^2) \, dy + (y + 2yz) \, dz)$ is independent of the path of integration from $(1, 2, 2)$ to $(2, 3, 4)$. Evaluate the integral.

14 a) Prove that eigen values of Hermitian matrix are real

ii) a skew-Hermitian matrix are zero or purely imaginary.

 b) Examine $A = \begin{bmatrix} 1 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 3 \end{bmatrix}$ is positive definite.

 15 a) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1.35.(2n-1)}{2.46..(2n)} x^n$

 b) Test the convergence of the series $1 + 3x + 5x^2 + 7x^3 + \dots$

16 a) State and prove Cayley Hamilton theorem.

b) Find the eigen values and the corresponding eigen vectors.

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

 17 a) If $x = r \cos \theta$, $y = r \sin \theta$; then find $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ (5)

 b) If $u = \log [x^2 + xy + y^2]$ then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$. (5)
