

FACULTY OF ENGINEERING & INFORMATION TECHNOLOGIES

B.E. 1 Year (New) (Common to all branches) (Main) Examination, June 2011

MATHEMATICS - I

Time : 3 Hours]

[Max. Marks : 75

Note : Answer all questions from Part - A. Answer any five questions from Part - B.

PART A

(Marks : 25)

- Using the Lagrange's mean value theorem, show that $\sin b - \sin a = \cos c(b - a)$ 2
- Find the envelope of the family of curves $y = 3cx - c^3$, c is a parameter. 3
3. If $f(x, y, z) = xyz^4$, $x = t^2$, $y = te^{-t}$, $z = te^{-t}$, find $\frac{df}{dt}$
4. Find the linear Taylor series polynomial approximation to the function $y = 2x^3 + 3y^3 - 4x^2y$ about the point $(1, 2)$.
5. If $\vec{r} = xi + yj + zk$, show that $(\nabla \cdot \vec{V})i = Ci$. 2
6. Find the directional derivative of the function $x, y, z = (1, 2, 3)$ in the direction of the line $i = j, j = i..$ 3
7. Find the values of X such that the rank of $A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & A & 5 \\ 4 & 8 & X \end{pmatrix}$ is 2.
8. Find the sum and the product of eigen values of the matrix $\begin{pmatrix} 10 & 0 & 8 \\ 4 & 9 & 6 \\ 2 & 7 & 5 \end{pmatrix}$
9. Find the values of x for which the series $\sum_{n=0}^{\infty} E(4)^n x^n$ is convergent.
10. Show that the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ converges absolutely.

PART B

(Marks 50)

11. (a) Find the radius of curvature of the curve $x = a(1 - \sin \theta)$, $y = a(1 - \cos \theta)$ at $\theta = \frac{\pi}{2}$ 5
- (b) Find the evolute of the curve $x = at^2$, $y = at^3$. 5

(This paper contains 2 pages)

P.T.O.

12. (a) Trace the curve $r = a(1 + \cos \theta)$. 6
- (b) Show that $f(x, y) = \frac{x^2 + y^2}{x - y}$ is not continuous at $(0, 0)$. 4
13. (a) Prove that $\text{curl}(fV) = (\text{grad } f) \times V + f \text{curl } V$. 5
- (b) Using Green's theorem, evaluate $\int_C (x^2 + y^2) dx + (y + 2x) dy$, where C is the boundary of the region bounded by the curves $y^2 = x$ and $x^2 = y$. 5
14. (a) If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, Show that $A^n = n^{-2} + A^2 - I$, $n \geq 3$ using Cayley-Hamilton theorem. 5
- (b) Reduce $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$ to the diagonal form. 5
15. Discuss the convergence of the series.
- (a) $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \dots$ 4
- (b) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ 6
16. (a) If $f(x, y) = x^2 + y^2$, compute $\frac{\partial^2 f}{\partial x^2}(0, 0)$. 4
- (b) Find the minimum value of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$. 6
17. (a) Evaluate $\int_0^1 \int_0^1 e^{-(21-y^2)} dx dy$. 5
- (b) Test whether the vectors $(1, 1, 0, 1)$, $(1, 1, 1, 1)$, $(4, 4, 1, 1)$, $(1, 0, 0, 1)$ are linearly independent or not 5