DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

SEMESTER EXAMINATION: MAY - 2017

Mechanical/Electrical/ExTC/Chemical/Petrochemical/Computer/IT/Civil

Subject : Engineering Mathematics-I (New) BSH101 **Semester** : I

Time : 03 Hrs **Max. Marks** : 60

2 MAY 2017

INSTRUCTION: ATTEMPT ANY FIVE QUESTIONS.

Q1. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 5 & 4 \end{bmatrix}$ by reducing it to normal form. [4 Marks]

(b) For what values of k is the following system of equations consistent, and hence solve for them: [4 Marks]

$$x + y + z = 1; x + 2y + 4z = k; x + 4y + 10z = k^2.$$

(c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. [4 Marks]

Q2. (a) Find the n^{th} derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ in terms of r and θ. [4 Marks]

(b) If $y = (x^2 - 1)^n$, prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n + 1)y_n = 0$. [4 Marks]

(c) Expand $f(x + h) = \tan^{-1}(x + h)$ in powers of h and hence find the value of $\tan^{-1}(1.003)$ upto five places of decimal. [4 Marks]

Q3. (a) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right]$. [4 Marks]

(b) If z is a homogeneous function of degree n in x, y, then prove that [4 Marks]

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$

(c) If $F = F(x, y, z)$ where $x = u + v + w$, $y = uv + vw + wu$, $z = xyz$, then show that [4 Marks]

$$u\frac{\partial F}{\partial u} + v\frac{\partial F}{\partial v} + w\frac{\partial F}{\partial w} = x\frac{\partial F}{\partial x} + 2y\frac{\partial F}{\partial y} + 3z\frac{\partial F}{\partial z}.$$

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Q4. (a) Expand $f(x, y) = \cos x \sin y$, as far as the terms of third degree. [4 Marks]

(b) If the sides and angles of a plane triangle vary in such a way that its circum-radius remains constant, prove that $\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} = 0$, where da, db, dc are smaller increments in the sides a, b, c respectively. [4 Marks]

(c) Find the maximum and minimum distances from the origin to the curve $3x^2 + 4xy + 6y^2 = 140$. [4 Marks]

Q5. (a) Change to polar co-ordinates and evaluate $I = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$. [4 Marks]

(b) Evaluate $I = \int_0^1 \int_{\sqrt{x}}^1 \frac{y^2}{\sqrt{y^4 - x^2}}\, dx dy$ by changing the order of integration. [4 Marks]

(c) Evaluate $I = \int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dy dx dz$. [4 Marks]

Q6. (a) Find the interval of convergence of the series $\sum \frac{n!}{n^n} x^n$. [4 Marks]

(b) Test the convergence of the series $\sum \frac{(n+1)^n}{n^{n+1}} x^n$. [4 Marks]

(c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{(n^n)^2}$. [4 Marks]

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