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Roll No:
Application No:
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Exam Date: **05-Oct-2020**
Exam Time: **15:00-18:00**
Examination: **1. Course Code - Ph.D.**
2. Field of Study - Mathematical Sciences (MATH)

SECTION 1 - PART I

Question No.1 (Question Id - 6)

The value of the integral $\frac{1}{2\pi i}\int_{|z|=1} \frac{2^{2^z}}{z^2}\, dz$ is equal to :

(A) log 2
(B) $(\log 2)^2$
(C) $2(\log 2)^2$ **(Correct Answer)**
(D) $\frac{(\log 2)^2}{2!}$

Question No.2 (Question Id - 1)

For each $n \in \mathbf{N}$, consider the map $g_n : [0, 3] \to \mathbf{R}$ given by $g_n(t) = \frac{t^n}{n!}$ for $t \in [0, 3]$. Which of the following assertions is **correct** ?

(A) $\{g_n\}$ converges to 0 uniformly on [0, 3]. **(Correct Answer)**
(B) $\{g_n\}$ converges to 0 pointwise on [0, 3] but not uniformly.
(C) $\{g_n\}$ converges to 0 pointwise on [0, 1] but not on [0, 3].
(D) $\{g_n\}$ does not converge pointwise to 0 on [0, 1].

Question No.3 (Question Id - 7)

Which of the following expressions defines a metric on **R** ?

(A) $d(x, y) := \sqrt{x^2 + y^2}$ for $x, y \in \mathbf{R}$.
(B) $d(x, y) := \sqrt{|x^2 - y^2|}$ for $x, y \in \mathbf{R}$.
(C) $d(x, y) := |x^3 - y^3|$ for $x, y \in \mathbf{R}$. **(Correct Answer)**
(D) $d(x, y) := |x^4 - y^4|$ for $x, y \in \mathbf{R}$.

Question No.4 (Question Id - 8)

For $1 \le p < \infty$, consider the normed spaces

$$l_p := \left\{(x_n) : \sum_n |x_n|^p < \infty\right\} \text{ and } c_0 := \left\{(x_n) : \lim_{n\to\infty} x_n = 0\right\}$$

Which of the following assertions is correct ?

(A) l_1 has a countable subset B such that span(B) = l_1
(B) $l_{10} \subset l_{20}$ and $c_0 \not\subseteq l_1$ **(Correct Answer)**
(C) $l_{20} \subset l_{10}$ and $c_0 \not\subseteq l_1$
(D) $c_0 \subseteq l_1$

Question No.5 (Question Id - 3)

Let V be a finite dimensional vector space over **R**. A subspace W of V is said to be invariant under a linear transformation T : V → V if T(W) ⊆ W. Suppose W_0 is a subspace of V such that W_0 is invariant under every linear transformation from V to itself. Which of the following is **true** ?

(A) ◯ dim $W_0 = 1$
(B) ◯ dim W_0 = dim V - 1
(C) ◯ **$W_0 = (0)$ (Correct Answer)**
(D) ◯ dim W_0 cannot be determined from the given information.

Question No.6 (Question Id - 2)

The set of limit points of the set $A = \left\{\frac{n}{4^k} : k \in \mathbf{N}, |n| < 4^k\right\}$ is :

(A) ◯ A itself
(B) ◯ A ∪ {-1, 1}
(C) ◯ (-1, 1)
(D) ◯ **[-1, 1] (Correct Answer)**

Question No.7 (Question Id - 9)
In a class of 60 students, 55 students register for Mathematics, 47 register for Physics and 34 students register for Chemistry. The minimum number of students who must have registered for all the three subjects is :

(A) ◯ 34
(B) ◯ 13
(C) ◯ **16 (Correct Answer)**
(D) ◯ 24

Question No.8 (Question Id - 10)
Consider the following statements :

A. There are 20 primitive roots modulo 25.

B. There are 8 primitive roots modulo 25.

C. There are 16 primitive roots modulo 100.

Which of the above statements is/are **correct** ?

(A) ◯ A only
(B) ◯ **B only (Correct Answer)**
(C) ◯ A and C only
(D) ◯ B and C only

Question No.9 (Question Id - 5)
Let F be a field having k ≥ 4 elements. Consider the following statements :

A. F contains more than 2 roots of 1.

B. F is isomorphic to $\mathbf{Z}/p^n\mathbf{Z}$ for some prime number p and n ϵ **N**.

C. F contains $\mathbf{Z}/p\mathbf{Z}$ for some prime number p.

Which of the above statements is/are necessarily **true** ?

(A) ◯ **A and C only (Correct Answer)**
(B) ◯ All A, B and C
(C) ◯ B and C only
(D) ◯ B only

Question No.10 (Question Id - 4)

Let α, β and γ be the eigenvalues of a matrix $A \in M_3(\mathbf{R})$ such that $A^3 - A^2 + 2I = 0$. Then the value of $\alpha^2 + \beta^2 + \gamma^2$ is :

(A) 5
(B) 3
(C) -5
(D) **1 (Correct Answer)**

SECTION 2 - PART II

Question No.1 (Question Id - 12)

The set $\{z \in \mathbf{C} : |e^z| = |z|\}$ is :

(A) empty
(B) a non-empty finite set
(C) a countably infinite set
(D) **an uncountable set (Correct Answer)**

Question No.2 (Question Id - 14)

Consider sets and operations :

$G_1 = \{f : \mathbf{R} \to \mathbf{R} | f$ is continuous$\}$ with respect to composition of maps and pointwise multiplication.

$G_2 = \{f : \mathbf{R} \to \mathbf{R} | f$ is continuous$\}$ with respect to pointwise addition and multiplication.

$G_3 = \{f : \mathbf{R}^2 \to \mathbf{R}^2 | f$ is a linear projection onto a one-dimensional subspace of $\mathbf{R}^2\}$ with respect to addition and composition.

$G_4 = \{f : \mathbf{R}^2 \to \mathbf{R}^2 | f$ is linear$\}$ with respect to addition and composition.

Which of the above is/are commutative ring(s) with unity ?

(A) G_3 only
(B) G_2 and G_4 only
(C) G_1, G_2 and G_4 only
(D) **G_2 only (Correct Answer)**

Question No.3 (Question Id - 24)

What is the remainder when 28! is divided by 31 ?

(A) 16
(B) **15 (Correct Answer)**
(C) 30
(D) 1

Question No.4 (Question Id - 22)

Let $(X, ||\cdot||)$ be a Banach space and $T : X \to X$ be a linear map. Define $||\cdot||_T : X \to [0, \infty)$ by $||x||_T = ||T(x)||$ for $x \in X$. Consider the following assertions :

A. $||\cdot||_T$ is a norm on X if and only if T is surjective.

B. $||\cdot||_T$ is a norm on X if and only if T is injective.

C. $||\cdot||_T$ is a norm on X if and only if T is continuous.

D. $(X, ||\cdot||_T)$ is Banach space if T is bijective.

Which of the above assertions is/are always **true** ?

(A) A and D only
(B) B and C only
(C) B only
(D) **B and D only (Correct Answer)**

Question No.5 (Question Id - 20)

Let X = **Z** and τ be the smallest topology on X containing all sets of the form {n, n + 3} for all n ϵ **Z**.
Consider the following assertions :

A. τ is same as the smallest topology on X containing all sets of the form {n, n + 1} for all n ϵ **Z**.

B. τ is same as the smallest topology on X containing all sets of the form {n, n + 2} for all n ϵ **Z**.

C. τ is same as the smallest topology on X containing all sets of the form {n, n + 1, n + 2} for all n ϵ **Z**.

D. τ is a countable collection.

Which of the above assertions is/are **correct** ?

(A) ◯ C only
(B) ◯ A and D only
(C) ◯ **A, B and C only (Correct Answer)**
(D) ◯ A, B and D only

Question No.6 (Question Id - 23)

For any fixed n ϵ **N**, the number of ordered triplets (X_1, X_2, X_3) of subsets of **N** such that $X_1 \cup X_2 \cup X_3 = \{1, 2, . . ., n\}$ is equal to :

(A) ◯ **7^n (Correct Answer)**
(B) ◯ 8^n
(C) ◯ n^8
(D) ◯ n^3

Question No.7 (Question Id - 19)

The set {z ϵ **C** : $e^z = z$} is :
(A) ◯ empty
(B) ◯ a non-empty finite set
(C) ◯ **a countably infinite set (Correct Answer)**
(D) ◯ an uncountable set

Question No.8 (Question Id - 18)

Let S = {A ϵ M_2(**R**)|A^2 = I}. Which of the following assertions is **true** ?

(A) ◯ **S is not a group. (Correct Answer)**
(B) ◯ S is a finite abelian group.
(C) ◯ S is an infinite abelian group.
(D) ◯ S is an infinite non-abelian group.

Question No.9 (Question Id - 13)

Let m denote the Lebesgue measure on **R**. Consider the following assertions :

A. If U is an open set in **R** containing **Q**, then m(U) = ∞.

B. There exists an open set U in **R** containing **Q** with $m(U) < \frac{1}{2020}$.

C. If U is an open set in **R** containing **Q** with m(U) = ∞, then m(**R**\U) = 0.

D. If G is a closed set in **R** containing **Q**, then m(G) = ∞.

Which of the above assertions are always **true** ?

(A) ◯ **B and D only (Correct Answer)**
(B) ◯ A, C and D only

(C) ○ A and D only
(D) ○ B, C and D only

Question No.10 (Question Id - 21)

Let $\{A_n : n \in \mathbf{N}\}$ be a countable collection of non-empty subsets of $\mathbf{R}^2$ such that $A_{n+1} \subseteq A_n$ for all $n \in$ **N**. Consider the following assertions :

A. If A_n is connected for every $n \in \mathbf{N}$, then $\cap_n A_n$ is connected.

B. If A_n is compact for every $n \in \mathbf{N}$, then $\cap_n A_n$ is compact.

C. If A_n is uncountable for every $n \in \mathbf{N}$, then $\cap_n A_n$ is uncountable.

D. If A_n is countable for every $n \in \mathbf{N}$, then $\cap_n A_n$ is non-empty.

Which of the above assertions is/are always **true** ?

(A) ○ **B only (Correct Answer)**
(B) ○ A and D only
(C) ○ C and D only
(D) ○ A and B only

Question No.11 (Question Id - 11)

Let S_1 and S_2 be the series $S_1 = \sum_{n=1}^{\infty} \frac{1}{2^{\log n}}$ and $S_2 = \sum_{n=1}^{\infty} \frac{(-1)^n \sin n}{\sqrt{n}}$

(A) ○ S_1 and S_2 both converge.
(B) ○ **S_1 diverges and S_2 converges. (Correct Answer)**
(C) ○ S_1 converges and S_2 diverges.
(D) ○ S_1 and S_2 both diverge.

Question No.12 (Question Id - 16)

Let V be a finite-dimensional vector space over **R**. Let $\{v_1, v_2, \ldots, v_n\}$ be a basis for V and let $\{w_1, w_2, \ldots, w_n\} \subset V$. Consider the following statements :

A. There exists a unique linear map $T : V \to V$ such that $T(v_i) = w_i$ for $1 \le i \le n$.

B. If there exists a linear map $T : V \to V$ such that $T(v_i) = w_i$ for $1 \le i \le n$, then T is injective.

C. If there exists an injective linear map $T : V \to V$ such that $T(v_i) = w_i$ for $1 \le i \le n$, then $\{w_1, w_2, \ldots, w_n\}$ is a basis for V.

D. There exists a unique linear map $T : V \to V$ such that $T(w_i) = v_i$ for $1 \le i \le n$.

Which of the above statements are **correct** ?

(A) ○ A and D only
(B) ○ A, B and C only
(C) ○ **A and C only (Correct Answer)**
(D) ○ A, B and D only

Question No.13 (Question Id - 15)

Let V, W and Z be finite-dimensional vector spaces over **R**. Let T : W → Z and S : V → W be linear maps. Consider the following statements :

A. If $T \cdot S$ is invertible then so are T and S.

B. If S and T are both injective then dim Z ≤ dim V .

C. If dim W > dim V , then $T \cdot S$ cannot be surjective.

D. If dim W < dim Z, then $T \cdot S$ cannot be surjective.

Which of the above statements is/are always **true** ?

(A) ○ **D only (Correct Answer)**
(B) ○ A and C only
(C) ○ B and D only
(D) ○ B only

Question No.14 (Question Id - 17)
Consider the following statements :

A. There exists a finitely generated group containing some element of infinite order.

B. There exists an infinite group which is not finitely generated but all whose elements have finite order.

C. There exists a finitely generated infinite group no element of which has infinite order.

Which of the above statements are **correct** ?

(A) ○ **All A, B and C (Correct Answer)**
(B) ○ B and C only
(C) ○ A and C only
(D) ○ A and B only

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