SECTION I - PART I

Question No.1 (Question Id - 4)

Which of the following is a compact subset of ℝ ?

(A) ○ $\left\{-\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\} \cup (0, \pi]$ **(Correct Answer)**

(B) ○ $\left\{-\frac{1}{n} : n \in \mathbb{N}\right\} \cup (0, 1]$.

(C) ○ $\{1, 2, 3\} \cup [4, 5] \cup \left\{6 + \frac{1}{n} : n \in \mathbb{N}\right\}$.

(D) ○ $\left\{1 - \frac{1}{n} : n \in \mathbb{N}\right\} \cup (1, 2]$.

Question No.2 (Question Id - 10)

Let G be an abelian group of order 16. Which of the following is **true** ?

(A) ○ There exists $g \in G$ such that order of g is 8.

(B) ○ If there exists a subgroup H of G of order 8, then there exists $g \in G$ with order 8.

(C) ○ If there exists $g \in G$ with order 8, then G is cyclic.

(D) ○ **There is a one-to-one group homomorphism $\varphi : G \to S_m$ for some m ≥ 1. (Correct Answer)**

Question No.3 (Question Id - 8)

Consider the system of linear equations :

$3x + y - z = \alpha$

$-x + 2y + 5z = \beta$

$4x + z = 7$

For which α and β does this system have a unique solution ?

(A) ○ For no α, β ∈ ℝ there is a unique solution.

(B) ○ α is unique but β can be arbitrary.

(C) ○ α and β are both unique.

(D) ○ **For all α, β ∈ ℝ there is a unique solution. (Correct Answer)**

Question No.4 (Question Id - 6)

What are the maximum and minimum of the function $f(x) = e^x - x$ on the interval $\left[-1, \frac{1}{2}\right]$?

(A) ○ $1 + \frac{1}{e}$ and $\sqrt{e} - \frac{1}{2}$.

(B) ○ $1 + \frac{1}{e}$ and 1. **(Correct Answer)**

(C) ○ $\sqrt{e} - \frac{1}{2}$ and 1.

(D) ○ $\sqrt{e} - \frac{1}{2}$ and $1 + \frac{1}{e}$.

Question No.5 (Question Id - 1)

Let X, Y and Z be finite sets and let $f : X \to Y$ and $g : Y \to Z$ be maps.

Which of the following assertions is always true ?

(A) ○ If gof is a bijection, then both g and f are bijections.

(B) ○ If g is one to one, then gof is also one to one.

(C) ○ If f is onto, then gof is also onto.

(D) ○ **If gof is onto, then |Z| ≤ |Y|, where |A| denotes the number of elements in any finite set A.**

(Correct Answer)

Question No.6 (Question Id - 9)

Question No.7 (Question Id - 7)

What is the value of the integral $\int_0^{\frac{\log 3}{2}} \frac{dx}{e^{-x}+e^{x}}$?

(A) $\frac{\pi}{12}$ **(Correct Answer)**

(B) $\log\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right) - \log 2$

(C) $\sqrt{3}+\frac{1}{\sqrt{3}} - 2$

(D) $\frac{\pi}{6}$

Question No.8 (Question Id - 3)

Let $a_n = (-1)^{n+1}\left(1+\frac{1}{2n+1}\right)$ for $n \geq 1$. Which of the following is **correct** ?

(A) **lim sup a_n = 1 and lim inf a_n = -1. (Correct Answer)**

(B) lim sup a_n = 1 and lim inf a_n = 1.

(C) lim sup a_n = -1 and lim inf a_n = -1.

(D) lim sup a_n = - 1 and lim inf a_n = 1.

Question No.9 (Question Id - 2)

Consider a series $\sum_{n=1}^{\infty} a_n$ of real numbers. Which of the following assertions is necessarily **true** ?

(A) If $|a_n| \leq \frac{n+1}{n^3}$ for all $n \geq 1$, then $\sum_n a_n$ converges conditionally but it does not necessarily converge absolutely.

(B) If $a_n \leq \frac{n+1}{n^3}$ for all $n \geq 1$, then $\sum_n a_n$ converges conditionally.

(C) If $-\frac{1}{n^2} \leq a_n \leq \frac{n+1}{n^3}$ for all $n \geq 1$, then $\sum_n a_n$ converges absolutely. **(Correct Answer)**

(D) If $0 \leq a_n \leq \frac{n^2+1}{n^3}$ for all $n \geq 1$, then $\sum_n a_n$ converges absolutely.

Question No.10 (Question Id - 5)

The sum $\frac{1}{1001}+\frac{1}{1002}+\ldots+\frac{1}{2000}$ is :

A. less than 1.

B. more than $\frac{1}{2}$.

C. more than log2.

D. less than log2.

Which of the above assertions are **correct** ?

(A) A, B and C only

(B) **A, B and D only (Correct Answer)**

B. If $\{x_n\}$ is bounded, then it contains a Cauchy subsequence.

C. If $\{x_n\}$ is unbounded, then it cannot contain a convergent subsequence.

D. If $\{x_n\}$ is bounded and a subsequence of $\{x_n\}$ converges to a real number L, then $\{x_n\}$ also converges to L.

Which of the above assertions is/are **true** ?

(A) A only
(B) **A and B only (Correct Answer)**
(C) B and C only
(D) D only

Question No.2 (Question Id - 23)

Consider the subset

$$N := \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R} \right\}$$

of the group of 2 × 2 matrices

$$G := \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a,\ b,\ d \in \mathbb{R},\ ad=1 \right\}$$

under matrix multiplication. Which of the following statements is correct ?

(A) N is not a subgroup of *G*.
(B) N is a subgroup of *G*, but N is not normal.
(C) N is a subgroup of *G* and the number of cosets of N in *G* is finite.
(D) **N is a subgroup of *G* and there are infinitely many cosets of N in *G*. (Correct Answer)**

Question No.3 (Question Id - 22)

Let G be a group in which every element other than identity has order 2. Then, which of the following statements is necessarily **true** ?

(A) *G* must be finite and abelian.
(B) ***G* can be infinite, but *G* must be abelian. (Correct Answer)**
(C) *G* is not necessarily abelian, but it must be finite.
(D) *G* may be non-abelian as well as infinite.

Question No.4 (Question Id - 18)

What is the area of the portion of the sphere $x^2 + y^2 + z^2 = R^2$ lying between the planes $z = R$ and $z = \frac{\sqrt{3}R}{2}$?

(A) $\pi R^2(2 - \sqrt{3})$ **(Correct Answer)**
(B) $\pi R^2 \sqrt{3}$
(C) $\frac{1}{2}\pi R^2$
(D) $\frac{1}{4}\pi R^2$

Question No.5 (Question Id - 21)

Let $A \in M_{4x3}(\mathbb{R})$, $B \in M_{3x4}(\mathbb{R})$ and $C \in M_{4x5}(\mathbb{R})$. Consider the following assertions :

A. The matrix ABC cannot have rank equal to 4.

B. AB can have rank 3 but BC cannot have rank 4.

C. ABC and BA can have ranks at most 3.

D. Rank of AB must be less than or equal to rank of BC.

What is the value of lim [illegible] ?

(A) - log2
(B) - 2 log2
(C) **1 - 2 log2 (Correct Answer)**
(D) 1 - 3 log2

Question No.7 (Question Id - 17)

A function f : (a, b) → ℝ is said to be uniformly continuous if for every ∈ > 0 there exists a δ > 0 such that $|f(x) - f(y)| < \epsilon$ whenever $|x - y| < \delta$ (and δ is independent of x and y).
Let f : (0, 1) → ℝ be the map given by $f(x) = \sqrt{x}$. Consider the following assertions :

A. f is differentiable on (0, 1).

B. f is differentiable and f' is bounded on (0, 1).

C. f is uniformly continuous on (0, 1).

D. f is differentiable and f' is uniformly continuous on (0, 1).

Which of the above assertions is/are **correct** ?

(A) A only
(B) **A and C only (Correct Answer)**
(C) A, B and C only
(D) A, B and D only

Question No.8 (Question Id - 13)

Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with real coefficients and radius of convergence R such that $0 < R < \infty$. Consider the following assertions :

(A) If $\sum_{n=0}^{\infty} a_n x^n$ converges for some x with $|x| = R$, then $\sum_{n=0}^{\infty} a_n x^n$ converges for every x with $|x| = R$.

(B) If $|x| > R$, then $\sup_{k \geq 10} \sum_{n=1}^{k} |a_n||x|^n = \infty$.

(C) If $\sum_{n=0}^{\infty} a_n x^n$ diverges for some x with $|x| = R$, then $\sum_{n=0}^{\infty} a_n x^n$ diverges for every x with $|x| = R$.

(D) Let $S_k(x) = \sum_{n=2}^{k} a_n x^n$ for all $k \geq 2$. Then, $\{S_k(x)\}_{k=2}^{\infty}$ is a Cauchy sequence for every $x \in (-R, R)$.

Which of the above assertions is/are correct ?

(A) A and C only
(B) **B and D only (Correct Answer)**
(C) D only
(D) B only

Question No.9 (Question Id - 19)

Let S be the sphere $x^2 + y^2 + z^2 = R^2$, **F** be the vector field on $\mathbb{R}^3$ given by $\mathbf{F} = x^3 \hat{i} + y^3 \hat{j} + z^3 \hat{k}$ and **n** denotes the unit normal vector to the surface S. What is the value of the surface integral $\iint_S F \cdot \mathbf{n} \, d\sigma$?

Consider the following assertions :

A. $\sup\{|x| \sin x : x \in \mathbb{R}\} = \infty$ and $\inf\{|x| \sin x : x \in \mathbb{R}\} = 0$

B. Let $f : \mathbb{R} \to \mathbb{R}$ be the map given by $f(x) = 2x + 3$. Then, $\sup\{f(\sin(x) + 5) : x \in \mathbb{R}\} = 15$ and $\inf\{f(\sin(x) + 5) : x \in \mathbb{R}\} = 11$

C. Let $f : \mathbb{R} \to \mathbb{R}$ be the map given by $f(x) = 5x + 5$. Then,

$$\inf\left\{f\left(\sin\left(\frac{1}{n}\right)\right) : n \geq 1\right\} = 0$$

D. Let $f : \mathbb{R} \to \mathbb{R}$ be the map given by $f(x) = 3x + 4$. Then, $\sup\{f(f(x)) : x \in (0, 2)\} = 34$

Which of the above assertions is/are **correct** ?

(A) ○ **B and D only (Correct Answer)**
(B) ○ B, C and D only
(C) ○ A, C and D only
(D) ○ A and D only

Question No.11 (Question Id - 11)

Let $X = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$, $Y = \{(x, y) \in \mathbb{R}^2 : x = y\}$ and $Z = \{(x, y) \in \mathbb{R}^2 : y = -x\}$. Consider the following assertions :

A. $X \cup Y \cup Z$ is an equivalence relation on $\mathbb{R}$.

B. $X \cup Y$ is a reflexive relation on $\mathbb{R}$ but not symmetric.

C. $X \cup Y$ is an equivalence relation on $\mathbb{R}$.

D. $Y \cup Z$ is an equivalence relation on $\mathbb{R}$.

Which of the above assertions are **correct** ?

(A) ○ A and B only
(B) ○ A, B and D only
(C) ○ **A and D only (Correct Answer)**
(D) ○ A, C and D only

Question No.12 (Question Id - 20)

Let V be a finite dimensional vector space over $\mathbb{R}$ with dim $V \geq 2$. Fix a non-zero vector $v_0 \in V$. Consider the following assertions :

A. There is a unique basis of V containing v_0.

B. There exist infinitely many bases of V containing v_0.

C. There is a unique injective linear map $T : V \to V$ such that $T(v_0) = v_0$.

D. There exist infinitely many linear isomorphisms $T : V \to V$ such that $T(v_0) = v_0$.

Which of the above assertions is/are **correct** ?

(A) ○ A only
(B) ○ C only
(C) ○ A and C only
(D) ○ **B and D only (Correct Answer)**

Question No.14 (Question Id - 24)

For which of the following *n* does *n*! have 2020 trailing zeros at the end ?

(A) ○ ***n* = 8097 (Correct Answer)**

(B) ○ *n* = 8085

(C) ○ *n* = 8080

(D) ○ *n* = 10100

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