

Roll No:

Application No:

Name:

 Exam Date: **05-Oct-2020**

 Exam Time: **09:00-12:00**

 Examination: **1. Course Code - Ph.D.**
**2. Field of Study - Physical Science (PHYH)**
**SECTION 1 - SECTION 1**
**Case Study - 1 to 3 (Question Id - 53)**

A sphere of radius  $R$  carries a polarization  $\vec{P}(\vec{r}) = k \vec{r}$  where  $k$  is a constant and  $\vec{r}$  is the vector from the center of the sphere. Answer the following three questions for this problem.

**Question No.1 (Question Id - 54)**

 The surface bound charge  $\sigma_b$  is :

- (A) ☐  $\frac{kr}{4\pi R^2}$
- (B) ☐  $\frac{1}{4\pi\epsilon_0} \frac{kr}{4\pi R^2}$
- (C) ☐  $kR \hat{r}$
- (D) ☒  **$kR$  (Correct Answer)**

**Question No.2 (Question Id - 55)**

 The volume bound charge ( $\rho_b$ ) is :

- (A) ☐  $\frac{1}{4\pi\epsilon_0} \frac{3k}{4\pi R^3}$
- (B) ☐  $-3kr$
- (C) ☒  **$-3k$  (Correct Answer)**
- (D) ☐  $9k^3 r^3 \hat{r}$

**Question No.3 (Question Id - 56)**

The electric field outside the sphere is :

- (A) ☐  $4\pi k R^2$
- (B) ☐  $\frac{4}{3} \pi k R^3 + 4\pi k R^2$
- (C) ☒  **$0$  (Correct Answer)**
- (D) ☐  $\frac{k}{3\epsilon_0} \vec{r}$

**Question No.4 (Question Id - 7)**

Consider the differential equation  $\frac{d^2 y}{dx^2} + \omega^2 y = 0$ . The solution of this equation can be expressed in the series form as :  $y(x) = \sum_{n=0}^{\infty} c_n x^n$ . Which of the following is the correct recursion relation for the coefficients of this series ?

- (A) ☒  $c_{n+2} = -\frac{\omega^2}{(n+2)(n+1)} c_n$  **(Correct Answer)**
- (B) ☐  $c_n = -\frac{\omega^2}{n(n+1)} c_{n+1}$
- (C) ☐  $c_n = \frac{\omega^2}{n(n-1)} c_{n-1}$
- (D) ☐  $c_{n+2} = \frac{\omega^2}{(n+2)(n+1)} c_n$

**Question No.5 (Question Id - 14)**

For an atom with an electronic configuration  $np^2$  (where  $n$  is the principal quantum number of a shell), the possible values of total angular momentum  $L$  and total spin  $S$  in the ground state are :

- (A) ☐  $L = 2$  and  $S = 0$   
 (B) ☐  $L = 2$  and  $S = 1$   
 (C) ☒  $L = 1$  and  $S = 1$  (Correct Answer)  
 (D) ☐  $L = 1$  and  $S = 0$

**Question No.6 (Question Id - 2)**

Which one of the following two-particle state  $\psi(\vec{r}_1, \vec{r}_2)$  correctly describes two identical bosons in the plane wave states given by the wave-vectors  $\vec{k}_1$  and  $\vec{k}_2$  ?

- (A) ☐  $\psi(\vec{r}_1, \vec{r}_2) = e^{i(\vec{k}_1 \cdot \vec{r}_1 + \vec{k}_2 \cdot \vec{r}_2)}$   
 (B) ☐  $\psi(\vec{r}_1, \vec{r}_2) = e^{i\vec{k}_1 \cdot \vec{r}_1} + e^{i\vec{k}_2 \cdot \vec{r}_2}$   
 (C) ☒  $\psi(\vec{r}_1, \vec{r}_2) = e^{i(\vec{k}_1 \cdot \vec{r}_1 + \vec{k}_2 \cdot \vec{r}_2)} + e^{i(\vec{k}_1 \cdot \vec{r}_2 + \vec{k}_2 \cdot \vec{r}_1)}$  (Correct Answer)  
 (D) ☐  $\psi(\vec{r}_1, \vec{r}_2) = e^{i(\vec{k}_1 \cdot \vec{r}_1 + \vec{k}_2 \cdot \vec{r}_2)} - e^{i(\vec{k}_1 \cdot \vec{r}_2 + \vec{k}_2 \cdot \vec{r}_1)}$

**Question No.7 (Question Id - 11)**

Electrons are ejected from calcium surface when monochromatic light of wavelength 488 nm falls on it. The work function of calcium is 2.28 eV. What is the maximum kinetic energy of the emitted electron ? (Planck's constant,  $h = 4.14 \times 10^{-15}$  eV·sec; speed of light,  $c = 3 \times 10^8$  m/sec)

- (A) ☐ 0.026 eV  
 (B) ☐ 26 eV  
 (C) ☐ 2.6 eV  
 (D) ☒ 0.26 eV (Correct Answer)

**Question No.8 (Question Id - 4)**

Which one of the following is **not true** about the superconductors ?

- (A) ☐ Type-II superconductors realize a mixed state between the critical magnetic fields  $H_{c1}$  and  $H_{c2}$ .  
 (B) ☐ In Type-I superconductors, the penetration depth ( $\lambda$ ) is smaller than the coherence length ( $\xi$ ).  
 (C) ☐ According to BCS theory, the copper pairs are formed due to electron-phonon interaction.  
 (D) ☒ Superconductivity is characterized by strongly paramagnetic behavior. (Correct Answer)

**Question No.9 (Question Id - 15)**

Consider a vector  $\vec{v} = x_1 \vec{a}_1 + x_2 \vec{a}_2 + x_3 \vec{a}_3$  in a real three dimensional vector space spanned by three basis vectors  $\vec{a}_1, \vec{a}_2$  and  $\vec{a}_3$ . Consider a new basis of three vectors :  $\vec{b}_1 = \vec{a}_1, \vec{b}_2 = \vec{a}_1 + \vec{a}_2$ , and  $\vec{b}_3 = \vec{a}_1 + \vec{a}_2 + \vec{a}_3$ . Let the vector  $\vec{v}$  given above be denoted in this new basis as :  $\vec{v} = y_1 \vec{b}_1 + y_2 \vec{b}_2 + y_3 \vec{b}_3$ . If the transformation matrix  $V$  between the components of the vector  $\vec{v}$  in the two bases is defined as :  $x_i = \sum_{j=1}^3 V_{ij} y_j$  for  $i = 1, 2, 3$ , then

- (A) ☐  $V = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(B) ☐

$$V = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad (\text{Correct Answer})$$

(C)  $\circ \quad V = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

(D)  $\circ \quad V = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

#### Question No.10 (Question Id - 5)

Which one of the following expressions is correct for the Helmholtz free energy  $F(T, V, N)$  of a thermodynamic system in canonical ensemble? Here,  $P$  is pressure,  $V$  is volume,  $N$  is the number of particles,  $\mu$  is chemical potential, and  $T$  is temperature.

- (A)  $\circ \quad F = -PV + \mu N$  (Correct Answer)  
 (B)  $\circ \quad F = PV + \mu N$   
 (C)  $\circ \quad F = -PV - \mu N$   
 (D)  $\circ \quad F = \mu N$

#### Question No.11 (Question Id - 1)

Let the angular momentum eigenstates with quantum number  $j$  be denoted as  $|j, m\rangle$ , where  $m = -j, -j + 1, \dots, j - 1, j$ . For a system of two angular momenta  $j_1$  and  $j_2$ , any state can be described as a linear superposition of their product states  $|j_1, m_1\rangle |j_2, m_2\rangle$ . For  $j_1 = 1$  and  $j_2 = 1/2$ , which of the following is the correct expression for the total angular momentum eigenstate with quantum numbers  $j_{\text{total}} = 3/2$  and  $m_{\text{total}} = 1/2$ ?

- (A)  $\circ \quad |j_{\text{total}} = 3/2, m_{\text{total}} = 1/2\rangle = \frac{1}{\sqrt{3}} (|1, 1\rangle |1/2, -1/2\rangle + \sqrt{2} |1, 0\rangle |1/2, 1/2\rangle)$  (Correct Answer)  
 (B)  $\circ \quad |j_{\text{total}} = 3/2, m_{\text{total}} = 1/2\rangle = \frac{1}{\sqrt{2}} (|1, 1\rangle |1/2, -1/2\rangle + |1, 0\rangle |1/2, 1/2\rangle)$   
 (C)  $\circ \quad |j_{\text{total}} = 3/2, m_{\text{total}} = 1/2\rangle = |1, 0\rangle |1/2, 1/2\rangle$   
 (D)  $\circ \quad |j_{\text{total}} = 3/2, m_{\text{total}} = 1/2\rangle = |1, 1\rangle |1/2, -1/2\rangle$

#### Case Study - 12 to 14 (Question Id - 33)

Consider a gas of  $N$  free electrons confined in a volume  $V$ . ( $m$  is the electron mass,  $\hbar$  is Planck's constant and  $k_B$  is Boltzmann's constant.)  
 Answer the following three questions on the free electron gas problem.

#### Question No.12 (Question Id - 34)

What is the density of states for the free electrons?

- (A)  $\circ \quad \frac{V}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{1/2} E^{3/2}$   
 (B)  $\circ \quad \frac{V}{2\pi^2} \left( \frac{2m}{\hbar^2} \right) E^{3/2}$   
 (C)  $\circ \quad \frac{V}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} E^{1/2}$  (Correct Answer)  
 (D)  $\circ \quad \frac{V}{2\pi^2} \left( \frac{2m}{\hbar^2} \right) E^{1/2}$

#### Question No.13 (Question Id - 35)

What is the Fermi energy in terms of  $N$  and  $V$ ?

- (A)  $\circ \quad \left( \frac{3\pi^2 N}{V} \right)^{2/3}$

(B) ☐ FirstRanker's choice

$$\frac{\hbar^2}{2m} \left( \frac{3\pi^2 N}{V} \right)^{\frac{2}{3}}$$

(C) ☐  $\frac{\hbar^2}{2m} \left( \frac{3\pi^2 N}{V} \right)^{\frac{2}{3}}$  (Correct Answer)

(D) ☐  $\left( \frac{3\pi^2 N}{V} \right)^{\frac{3}{2}}$

**Question No.14 (Question Id - 36)**

How does the specific heat ( $C_V$ ) of free electron gas vary with temperature ( $T$ ) at low temperatures ?

(A) ☐  $C_V \propto T^3$

(B) ☐  $C_V \propto e^{-\frac{\Delta}{k_B T}}$ , where  $\Delta$  is the energy gap

(C) ☐  $C_V \propto T^2$

(D) ☐  $C_V \propto T$  (Correct Answer)

**Case Study - 15 to 17 (Question Id - 45)**

Consider the function  $f(z) = e^{1/z}$  of a complex variable  $z = x + jy$  in a complex plane. Answer the following three questions on this function.

**Question No.15 (Question Id - 46)**

The function  $f(z) = e^{1/z}$  has :

(A) ☐ no singularity at  $z = 0$

(B) ☐ an essential singularity at  $z = 0$  (Correct Answer)

(C) ☐ a simple pole at  $z = 0$

(D) ☐ a branch point at  $z = 0$

**Question No.16 (Question Id - 47)**

Evaluate the integral  $\oint dz e^{1/z}$  over the closed contour given by the unit circle  $|z| = 1$  centered around the origin of the complex plane.

(A) ☐  $\pi$

(B) ☐  $i\pi$

(C) ☐  $i2\pi$  (Correct Answer)

(D) ☐  $2\pi$

**Question No.17 (Question Id - 48)**

The equation of the contour corresponding to a fixed value,  $A$ , of the amplitude of the function  $e^{1/z}$  is :

(A) ☐  $\left( x - \frac{1}{2\ln A} \right)^2 + y^2 = \frac{1}{4(\ln A)^2}$  (Correct Answer)

(B) ☐  $\left( x + \frac{1}{2\ln A} \right)^2 + y^2 = \frac{1}{4(\ln A)^2}$

(C) ☐  $\left( x - \frac{1}{\ln A} \right)^2 + y^2 = \frac{1}{(\ln A)^2}$

(D) ☐  $\left( x + \frac{1}{\ln A} \right)^2 + y^2 = \frac{1}{(\ln A)^2}$

**Question No.18 (Question Id - 19)**

For a classical system described by a pair of canonical coordinate  $q$  and momentum  $p$ , consider the transformation

$Q = -\sqrt{2p} \cos q$  and  $P = \sqrt{2p} \sin q$ . The Poisson bracket of the new variables  $Q$  and  $P$  is equal to :

- (A) ☐  $-\cos 2q$   
 (B) ☐  $\cos 2q$   
 (C) ☒ **1 (Correct Answer)**  
 (D) ☐ 0

**Case Study - 19 to 21 (Question Id - 41)**

Answer the following three questions on the relativistic corrections to the hydrogen problem.

**Question No.19 (Question Id - 42)**

The leading relativistic correction to the kinetic energy term in the hydrogen atom Hamiltonian is :

- (A) ☐  $\frac{p^4}{8m^3c^2}$   
 (B) ☐  $-\frac{p^3}{8m^3c^2}$   
 (C) ☒  $-\frac{p^4}{8m^3c^2}$  **(Correct Answer)**  
 (D) ☐  $\frac{p^5}{8m^3c^2}$

**Question No.20 (Question Id - 43)**

The relativistic correction to the hydrogen atom problem leading to spin-orbit interaction is given by :

- (A) ☐  $\xi(r) \vec{L} \cdot \vec{S}$ , where  $\xi(r) \propto r$   
 (B) ☒  $\xi(r) \vec{L} \cdot \vec{S}$ , where  $\xi(r) \propto r^{-3}$  **(Correct Answer)**  
 (C) ☐  $\xi(r) \vec{L} \cdot \vec{S}$ , where  $\xi(r) \propto r^{-2}$   
 (D) ☐  $\xi(r) \vec{L} \cdot \vec{S}$ , where  $\xi(r) \propto r^{-1}$

**Question No.21 (Question Id - 44)**

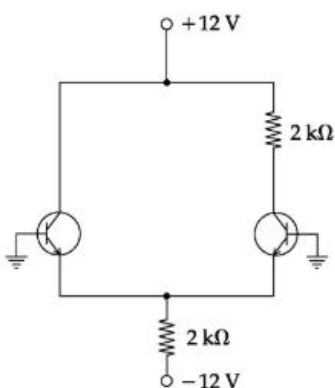
The relativistic correction due to Darwin to the hydrogen atom problem is given by  $\frac{1}{8\epsilon_0} \left( \frac{\hbar e}{mc} \right)^2 \delta(\vec{r})$  where  $\delta(\vec{r})$  is

Dirac delta function. Which of the following atomic states will be affected by the Darwin correction term ?

- (A) ☒ **Only  $l = 0$  states (Correct Answer)**  
 (B) ☐ Only  $l = 1$  states  
 (C) ☐ Only  $l = 2$  states  
 (D) ☐ All  $l$  states

**Case Study - 22 to 24 (Question Id - 37)**

For a single ended differential amplifier as given in the figure, answer the following **three** questions.



**Question No.22 (Question Id - 38)**

The tail current is :

- (A) ☐ 5 mA  
 (B) ☐ 10 mA  
 (C) ☒ 6 mA (Correct Answer)  
 (D) ☐ 8 mA

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**Question No.23 (Question Id - 39)**

The value of emitter current is :

- (A) ☐ 1 mA  
 (B) ☐ 2 mA  
 (C) ☒ 3 mA (Correct Answer)  
 (D) ☐ 4 mA

**Question No.24 (Question Id - 40)**

The value of the collector voltage :

- (A) ☐ 4 V  
 (B) ☒ 6 V (Correct Answer)  
 (C) ☐ 8 V  
 (D) ☐ 10 V

**Question No.25 (Question Id - 10)**

Which one of the following elements cannot be used as dopants in Silicon to make it n-type semiconductor ?

- (A) ☐ Arsenic  
 (B) ☐ Phosphorus  
 (C) ☒ Boron (Correct Answer)  
 (D) ☐ Antimony

**Question No.26 (Question Id - 13)**

Consider a particle in a state given by the wavefunction  $\Psi(x, y, z) = (y + iz)^2$ . This wavefunction is an eigenfunction of the angular momentum operator  $L_x$  with eigenvalue.

- (A) ☐  $-2\hbar$   
 (B) ☐  $-\hbar$   
 (C) ☐  $+\hbar$   
 (D) ☒  $+2\hbar$  (Correct Answer)

**Question No.27 (Question Id - 20)**

By doing an elastic scattering experiment with a beam of electrons of momentum  $p \approx 120 \text{ MeV}/c$ , we can determine :

(Planck's constant,  $h = 6.63 \times 10^{-34} \text{ J.s}$ ; speed of light,  $c = 3 \times 10^8 \text{ m/s}$ ; electron charge,  $e = 1.6 \times 10^{-19} \text{ C}$ )

- (A) ☐ the size of a biomolecule  
 (B) ☐ the lattice constants of a crystal of gold  
 (C) ☒ the size of an atomic nucleus (Correct Answer)  
 (D) ☐ none of the above

**Case Study - 28 to 30 (Question Id - 21)**

A "two-level" atom is considered to have only two energy levels with energies 0 and  $\epsilon$ . For a system of  $N$  non-interacting two-level atoms with total energy  $E$ , answer the following three questions.

**Question No.28 (Question Id - 22)**

What is the number of microstates  $\Omega(N, E)$  ?

- (A) ☐

- ☐ (A)  $\frac{N!}{\left(N + \frac{E}{\epsilon}\right)! \left(\frac{E}{\epsilon}\right)!}$   
☐ (B)  $\frac{N!}{\left(N - \frac{E}{\epsilon}\right)! \left(\frac{E}{\epsilon}\right)!}$  (Correct Answer)  
☐ (C)  $\frac{N!}{\left(N - \frac{E}{\epsilon}\right)! \left(N + \frac{E}{\epsilon}\right)!}$   
☐ (D)  $\frac{N!}{\left(N - \frac{\epsilon}{E}\right)! \left(\frac{\epsilon}{E}\right)!}$

**Question No.29 (Question Id - 23)**

What is the entropy per particle in the limit of large N ?

- ☐ (A)  $-k_B \left[ \left(1 - \frac{E}{N\epsilon}\right) \ln \left(1 - \frac{E}{N\epsilon}\right) - \left(\frac{E}{N\epsilon}\right) \ln \left(\frac{E}{N\epsilon}\right) \right]$   
☐ (B)  $+k_B \left[ \left(1 - \frac{E}{N\epsilon}\right) \ln \left(1 - \frac{E}{N\epsilon}\right) + \left(\frac{E}{N\epsilon}\right) \ln \left(\frac{E}{N\epsilon}\right) \right]$   
☐ (C)  $-k_B \left[ \left(1 - \frac{E}{N\epsilon}\right) \ln \left(1 - \frac{E}{N\epsilon}\right) + \left(\frac{E}{N\epsilon}\right) \ln \left(\frac{E}{N\epsilon}\right) \right]$  (Correct Answer)  
☐ (D)  $+k_B \left[ \left(1 + \frac{E}{N\epsilon}\right) \ln \left(1 + \frac{E}{N\epsilon}\right) - \left(\frac{E}{N\epsilon}\right) \ln \left(\frac{E}{N\epsilon}\right) \right]$

**Question No.30 (Question Id - 24)**

What is the corresponding temperature T ?

- ☐ (A)  $\frac{1}{T} = \frac{k_B}{\epsilon} \ln \left( \frac{N\epsilon}{E} - 1 \right)$  (Correct Answer)  
☐ (B)  $\frac{1}{T} = \frac{k_B}{\epsilon} \ln \left( \frac{N\epsilon}{E} + 1 \right)$   
☐ (C)  $\frac{1}{T} = \frac{k_B}{\epsilon} \ln \left( \frac{E}{N\epsilon} + 1 \right)$   
☐ (D)  $\frac{1}{T} = \frac{k_B}{\epsilon} \ln \left( \frac{E}{N\epsilon} - 1 \right)$

**Question No.31 (Question Id - 8)**

The decay  $n \rightarrow p + e^-$  of a neutron (n) into a proton (p) and an electron ( $e^-$ ) is forbidden due to the violation of conservation of :

- ☐ (A) Angular momentum and baryon number  
☐ (B) Energy and lepton number  
☐ (C) Angular momentum and lepton number (Correct Answer)  
☐ (D) Electric charge and baryon number

**Case Study - 32 to 34 (Question Id - 49)**

Consider a crystalline material which, under ambient conditions, is given to have the FCC (face-centered cubic) lattice structure with monoatomic basis. Answer the following three questions for this system.

**Question No.32 (Question Id - 50)**

A primitive unit cell of the monoatomic FCC crystal contains :

- ☐ (A) 1 atom (Correct Answer)  
☐ (B) 2 atoms  
☐ (C) 3 atoms  
☐ (D) 4 atoms



The phonon dispersion of a monoatomic FCC crystal has :

- (A) ☒ **3 branches of acoustic phonons only. (Correct Answer)**  
 (B) ☐ 3 branches of acoustic phonons, and 9 branches of optical phonons.  
 (C) ☐ 1 branch of acoustic phonons, and 2 branches of optical phonons.  
 (D) ☐ 3 branches of optical phonons only.

#### Question No.34 (Question Id - 52)

Suppose by changing the temperature, if the crystal structure of the material changes from the monoatomic FCC to monoatomic BCC (body-centered cubic), then the number of optical phonon branches will change by :

- (A) ☒ **0 (Correct Answer)**  
 (B) ☐ 2  
 (C) ☐ 3  
 (D) ☐ 6

#### Case Study - 35 to 37 (Question Id - 57)

Answer the following three questions on the semi-empirical formula for the binding energy of atomic nuclei in terms of the nuclear mass number A and the proton number Z.

#### Question No.35 (Question Id - 58)

In the formula for binding energy per nucleon, the volume energy term is :

- (A) ☒ **a constant (Correct Answer)**  
 (B) ☐ proportional to Z  
 (C) ☐ proportional to A  
 (D) ☐ proportional to  $A^{1/3}$

#### Question No.36 (Question Id - 59)

In the formula for binding energy per nucleon, the contribution from Coulomb repulsion between protons is :

- (A) ☐ proportional to Z only  
 (B) ☐ proportional to  $Z(Z-1)$  only  
 (C) ☐ proportional to  $Z(Z-1)A^{-1/3}$   
 (D) ☒ **proportional to  $Z(Z-1)A^{-4/3}$  (Correct Answer)**

#### Question No.37 (Question Id - 60)

In the formula for binding energy per nucleon, the pairing energy term is :

- (A) ☐ always zero  
 (B) ☒ **zero only when A is an odd integer (Correct Answer)**  
 (C) ☐ non-zero only when A is an odd integer  
 (D) ☐ always non-zero

#### Question No.38 (Question Id - 3)

If the scalar and vector potentials are given by  $\phi(\vec{r}, t) = 0$  and  $\vec{A}(\vec{r}, t) = -\frac{1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r}$ , the

corresponding electric field ( $\vec{E}$ ) is :

- (A) ☐ 0  
 (B) ☒  **$\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$  (Correct Answer)**  
 (C) ☐  $\frac{1}{4\pi\epsilon_0} \frac{q}{r} \hat{r}$   
 (D) ☐  $-\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$



A body of mass  $m$  is thrown up vertically with an initial speed  $u$ . The air exerts a drag force  $-kv$  upon it, where  $v$  is the instantaneous velocity of the body and  $k$  is a constant. The body also experiences gravitational acceleration  $g$ . Answer the following three questions on this problem.

**Question No.39 (Question Id - 30)**

What is the terminal speed attained by the body ?

- (A) ☒  $\frac{mg}{k}$  (Correct Answer)
- (B) ☐  $\frac{g}{k}$
- (C) ☐  $\frac{k}{mg}$
- (D) ☐  $u$

**Question No.40 (Question Id - 31)**

What is the time it will take to attain the maximum height ?

- (A) ☐  $\ln \left( 1 + \frac{mg}{ku} \right)$
- (B) ☐  $\frac{k}{m} \ln \left( 1 + \frac{ku}{mg} \right)$
- (C) ☒  $\frac{m}{k} \ln \left( 1 + \frac{ku}{mg} \right)$  (Correct Answer)
- (D) ☐  $\frac{m}{k} \ln \left( 1 + \frac{mg}{ku} \right)$

**Question No.41 (Question Id - 32)**

What is the maximum height attained by the body ?

- (A) ☐  $\frac{mu}{k} + g \left( \frac{m}{k} \right)^2 \ln \left( 1 + \frac{ku}{mg} \right)$
- (B) ☒  $\frac{mu}{k} - g \left( \frac{m}{k} \right)^2 \ln \left( 1 + \frac{ku}{mg} \right)$  (Correct Answer)
- (C) ☐  $\frac{mu}{k} - g \left( \frac{m}{k} \right)^2 \ln \left( 1 - \frac{ku}{mg} \right)$
- (D) ☐  $\frac{mu}{k} + g \left( \frac{m}{k} \right)^2 \ln \left( 1 - \frac{ku}{mg} \right)$

**Question No.42 (Question Id - 16)**

The Fourier transformation for a function  $f(x)$  of a real variable  $x$  can be defined as :

$f(x) = \int_{-\infty}^{+\infty} dk e^{ikx} g(k)$ , where  $g(k)$  is a function of another real variable  $k$ . If  $g(k) = e^{iky}$  for a given  $y$ , then what is  $f(x)$  ?

- (A) ☐  $\delta(x + y)$
- (B) ☐  $\delta(x - y)$
- (C) ☒  $2\pi\delta(x + y)$  (Correct Answer)
- (D) ☐  $2\pi\delta(x - y)$

**Question No.43 (Question Id - 17)**

In spectroscopy, the selection rule for transition between the rotational energy levels of a diatomic molecule (given by the rotational quantum number  $J$ ) states that the transition between two rotational levels is allowed if :

- (A) ☒  $\Delta J = \pm 1$  (Correct Answer)
- (B) ☐  $\Delta J = \pm 2$
- (C) ☐  $\Delta J = \pm 3$

(D) ☐ None of the above

**Question No.44 (Question Id - 9)**

For a classical system described by the Hamiltonian  $H(q, p)$  in terms of the generalized coordinates  $q$  and  $p$ , the Hamilton's equations of motion (in the standard notation) are :

(A) ☐  $\dot{q} = \frac{\partial H}{\partial p}$

$\dot{p} = \frac{\partial H}{\partial q}$

(B) ☐  $\dot{q} = -\frac{\partial H}{\partial p}$

$\dot{p} = -\frac{\partial H}{\partial q}$

(C) ☒  $\dot{q} = \frac{\partial H}{\partial p}$

$\dot{p} = -\frac{\partial H}{\partial q}$

(Correct Answer)

(D) ☐  $\dot{q} = -\frac{\partial H}{\partial p}$

$\dot{p} = \frac{\partial H}{\partial q}$

**Question No.45 (Question Id - 6)**

For a thermodynamic system of  $N$  particles at temperature  $T$ , which of the following relation is correct for the change in entropy  $S$  with respect to volume  $V$  ?

(A) ☐  $\left(\frac{\partial S}{\partial V}\right)_{T, N} = -\left(\frac{\partial P}{\partial T}\right)_{V, N}$

(B) ☒  $\left(\frac{\partial S}{\partial V}\right)_{T, N} = \left(\frac{\partial P}{\partial T}\right)_{V, N}$

(Correct Answer)

(C) ☐  $\left(\frac{\partial S}{\partial V}\right)_{T, N} = \left(\frac{\partial T}{\partial P}\right)_{S, N}$

(D) ☐  $\left(\frac{\partial S}{\partial V}\right)_{T, N} = -\left(\frac{\partial T}{\partial P}\right)_{S, N}$

**Question No.46 (Question Id - 12)**

A spin-1/2 particle in a magnetic field  $B$  pointing along  $y$ -direction is described by the Hamiltonian  $H = \mu_B B \sigma_y$ , where  $\sigma_y$  is the Pauli matrix corresponding to the  $y$  component of the spin-1/2 operator (and  $\mu_B$  is the Bohr magneton). For this system, the time evolution operator  $e^{-iHt/\hbar}$  can be written as :

(A) ☐  $\begin{bmatrix} \cos\left(\frac{\mu_B B t}{\hbar}\right) & -\sin\left(\frac{\mu_B B t}{\hbar}\right) \\ -\sin\left(\frac{\mu_B B t}{\hbar}\right) & \cos\left(\frac{\mu_B B t}{\hbar}\right) \end{bmatrix}$

(B) ☐  $\begin{bmatrix} \cos\left(\frac{\mu_B B t}{\hbar}\right) & i\sin\left(\frac{\mu_B B t}{\hbar}\right) \\ -i\sin\left(\frac{\mu_B B t}{\hbar}\right) & \cos\left(\frac{\mu_B B t}{\hbar}\right) \end{bmatrix}$

(C) ☐  $\begin{bmatrix} \cos\left(\frac{\mu_B B t}{\hbar}\right) & \sin\left(\frac{\mu_B B t}{\hbar}\right) \\ \sin\left(\frac{\mu_B B t}{\hbar}\right) & \cos\left(\frac{\mu_B B t}{\hbar}\right) \end{bmatrix}$

(D) ☒  $\begin{bmatrix} \cos\left(\frac{\mu_B B t}{\hbar}\right) & -\sin\left(\frac{\mu_B B t}{\hbar}\right) \\ \sin\left(\frac{\mu_B B t}{\hbar}\right) & \cos\left(\frac{\mu_B B t}{\hbar}\right) \end{bmatrix}$

(Correct Answer)

**Case Study - 47 to 49 (Question Id - 25)**

Consider the one-dimensional simple harmonic oscillator of mass  $m$  and frequency  $\omega$  described by the Hamiltonian,  $H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 x^2 = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$ , with eigenvalues  $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$  and the eigenstates  $|n\rangle$ . The creation and annihilation operators  $a^\dagger$  and  $a$  are related to the coordinate  $x$  and momentum  $p$  as :  $x = \sqrt{\frac{\hbar}{2m\omega}}(a^\dagger + a)$  and  $p = i\sqrt{\frac{m\hbar\omega}{2}}(a^\dagger - a)$ . Answer the following three questions on this problem.

**Question No.47 (Question Id - 26)**

The commutator  $[a^\dagger a, a^\dagger a^\dagger]$  is equal to :

- (A) ☐  $-2a^\dagger a^\dagger$   
 (B) ☐  $2a^\dagger a$   
 (C) ☒  $2a^\dagger a^\dagger$  (Correct Answer)  
 (D) ☐  $-2a^\dagger a$

**Question No.48 (Question Id - 27)**

What is the uncertainty in position,  $\sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ , in the eigenstate  $|n\rangle$  ?

- (A) ☐  $\sqrt{\frac{\hbar}{m\omega}}(2n+1)$   
 (B) ☒  $\sqrt{\frac{\hbar}{m\omega}}\left(n + \frac{1}{2}\right)$  (Correct Answer)  
 (C) ☐ 0  
 (D) ☐  $\sqrt{\frac{\hbar}{2}}$

**Question No.49 (Question Id - 28)**

Which of the following is the correct expression for the creation operator ?

- (A) ☐  $\sqrt{n+1}|n\rangle\langle n+1|$   
 (B) ☒  $\sum_{n=0}^{\infty} \sqrt{n+1}|n+1\rangle\langle n|$  (Correct Answer)  
 (C) ☐  $\sum_{n=0}^{\infty} \sqrt{n}|n\rangle\langle n+1|$   
 (D) ☐  $\sqrt{n}|n\rangle\langle n-1|$

**Question No.50 (Question Id - 18)**

Consider a rectangular waveguide with a cross-section of dimension 2 cm x 1 cm. If the driving frequency is  $1.7 \times 10^{10}$  Hz, the Transverse Electric (TE) mode that will propagate in this wave guide is :

- (A) ☐  $0.53 \times 10^{10}$  Hz  
 (B) ☒  $0.75 \times 10^{10}$  Hz (Correct Answer)  
 (C) ☐  $1.9 \times 10^{10}$  Hz  
 (D) ☐  $1.4 \times 10^9$  Hz

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