

[illegible]

- Explain how the vertical line test is used to detect functions.
- Graph the parabola  $f(x) = x^2$ . Explain why the secant lines between the points  $(-a, f(-a))$  and  $(a, f(a))$  have zero slope.
- Evaluate  $\lim_{x \rightarrow a} \frac{x-a}{\sqrt{x}-\sqrt{a}}, a > 0$ .
- Suppose  $h(x) = \begin{cases} 3x+b, & x \leq 2 \\ x-2, & x > 2 \end{cases}$ . Determine a value of the constant  $b$  for which  $\lim_{x \rightarrow 2} h(x)$  exists and state the value of the limit, if possible.
- Discuss the continuity of  $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$  at  $x = 0$ .
- Use the definition of the derivative to determine  $\frac{d}{dx}(\sqrt{x})$ . Also find the equation of the line tangent to the graph  $\sqrt{x}$  at  $(4, 2)$ .

- g) Compute the derivative of  $\frac{(x^2+3x-10)e^{5x}}{5x-1}$ .
- h) Find  $y'(x)$ , when  $\sin xy = x^2 + y$ .
- i) State second derivative test for local extrema.
- j) Suppose an airline policy states that all baggage must be box-shaped with a sum of length, width and height not exceeding 64 in. What are the dimensions and volume of a square-based box with the greatest volume under these conditions?

### SECTION-B

2. a) Find the inverse of the function  $f(x) = x^2 + 4$  and write it in the form  $y = f^{-1}(x)$ . Also verify the relationships  $f(f^{-1}(x)) = x$  and  $f^{-1}(f(x)) = x$ .
- b) Find the domain and range of the function  $f(x) = x^5 + \sqrt{x}$ .
3. a) Let  $f(x) = \begin{cases} -2x+4 & \text{if } x \leq 1 \\ \sqrt{x-1} & \text{if } x > 1 \end{cases}$ . Find the values of  $\lim_{x \rightarrow 1^-} f(x)$ ,  $\lim_{x \rightarrow 1^+} f(x)$ , and  $\lim_{x \rightarrow 1} f(x)$ , or state that they do not exist.
- (b) Evaluate  $\lim_{x \rightarrow \infty} \left( 5 + \frac{\sin x}{\sqrt{x}} \right)$ .
4. a) A particle moves along the curve  $6y = x^3 + 2$ . Find the points on the curve at which the  $y$ -coordinate is changing 8 times as fast as the  $x$ -coordinate.
- b) Prove that the function  $f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0 \end{cases}$  is everywhere continuous.
5. a) Find the derivative of  $\frac{\sin x + \cos x}{\sin x - \cos x}$ .
- b) Find the maximum value of  $2x^3 - 24x + 107$  in the interval  $[1, 3]$ .

## SECTION-C

6. a) If  $f(x) = \frac{(x^3 - 1)^4 \sqrt{3x - 1}}{x^2 + 4}$ . Find  $f'(x)$ . (3)
- b) A swimming pool is 50m long and 20m wide. Its depth decreases linearly along the length from 3m to 1m. It is initially empty and is filled at the rate of  $1\text{m}^3/\text{minute}$ . How fast is the water level rising 250 minutes after the filling begins? How long will it take to fill the pool? (5)
7. An 8-foot-tall fence runs parallel to the side of a house 3 feet away. What is the length of the shortest ladder that clears the fence and reaches the house? Assume that the vertical wall of the house and the horizontal ground have infinite extent. (8)
8. a) Find the anti derivative of the function  $\frac{4x^4 - 6x^2 + \sqrt{x}}{x}$ . Check your answer by taking derivative. (4)
- b) If  $\lim_{x \rightarrow 0} \frac{\sin 2x + a \sin x}{x^3}$  be finite, find the value of 'a' and the limit. (4)
9. Use the graphing guidelines to graph the function  $f(x) = \frac{10x^3}{x^2 - 1}$ . (8)

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